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Growing Old Gracefully:
Age-Phasing, Targets and Saving Rules*

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1. Introduction

Should people reduce their exposure to risky securities as they age? Personal finance experts frequently urge them to do so, but financial economists have a hard time coming up with a theoretical justification for this advice. Starting from the idea in Samuelson (1963), developed into the rigorous Merton (1969) result, that standard preference maximisation yields a constant proportion in risky rule, they puzzle about how to get to age related variability.[1]

The intuition underlying risk constancy is that we seek to optimise by trading off risk at the margin much as we trade off other commodities. We drive slower in wet weather to expose ourselves to constant risk while driving. Equally, so long as risky does not change its riskiness, we should not alter our portfolios.

In the Fall 1994 issue of this journal's stablemate publication, the *Journal of Portfolio Management*, Paul Samuelson succinctly surveys various arguments for age related reduction in equity exposure. The two most convincing are well articulated by Bodie et al (1992). First, if human capital is relatively safe, then as the stock of human capital depletes with age, a compensating financial portfolio adjustment should take place to preserve constant exposure to risk overall. This will have the effect of increasing the proportion of safe assets in a financial portfolio with age.

The second argument, less analytically crisp but perhaps more robust, is that people's ability to adjust their behaviour to accommodate the consequences of bad luck decreases with age. A family that takes a hit when the household head is in his 30s, or even 40s, has more margins on which to adjust than one whose head is approaching retirement. Work effort (including secondary labour force participation), job retraining, even family size, are all examples of adjustments that are much more feasible earlier rather than later in the life cycle.

Another case for an age-phased reduction in exposure to risky investments, due to Samuelson (1991), arises from how one views movements in equity prices. Equity prices can be characterised as mean-reverting or as following a random walk. If they are mean-reverting, then excellent returns on Wall Street will tend to be followed by weak returns, and vice versa. For a large class of attitudes toward risk, if you believe equity prices are mean-reverting then you will be much happier to invest in risky stocks when young, reducing your exposure as you age, as compared to a similar investor who views equity returns as a random walk.[2] Such an investor, with the same attitude towards risk as you, would maintain a constant exposure to risky stocks over his or her life cycle. Intuitively, this greater tolerance for risk when young on your part arises because of your belief in the odds that prices will rebound from a low level, and so risky investment isn't that risky after all.

Finally, Samuelson also gives play to the idea that a preference function embodying a minimum target level of retirement wealth will, independently of the above considerations, imply an optimal investment strategy involving age related reductions in equity exposure. Such a strategy will involve a safe sinking, or escrow, fund, put aside and invested in safe assets to ensure that the target is achieved. Drawing on an earlier paper (Samuelson 1989), he reports "definite and reasonable results … when people insist on the apparent importance of a minimum (retirement wealth) attainment." (Samuelson 1994, p.18). He argues that because the escrow fund must increase with age to meet its retirement target, and so long as there is enough initial wealth to meet the initial escrow requirement and have some over to invest in constant proportions – "à la Bernoulli" – age-phasing results.

In this he is mistaken. Formally, the error is simply an interpretative slip in his 1989 mathematical analysis. His intuitive 1994 account of the "escrow" argument for age phasing is misleading because it ignores the impact of the differential between risky and safe rates of return on the size of the overall accumulation.

This note does two things. First, we correct Samuelson's "escrow" argument, and explain with a numerical simulation that far from age-phasing holding in general, the opposite is more likely to be true – escrow is likely to lead to an investment strategy in which the proportion in risky *increases* over the life cycle. The age-phasing result will be generated in only a small minority of cases – those where, over a life cycle, safe does better than risky. Second, we demonstrate that for most of us, who accumulate our assets through the working phase of life, financial age-phasing may be a consequence of a saving rule. Financial planners who urge age-phasing may be advocating constant equity exposure after adjusting for the financial portfolio effects of saving through time.

2. The Impact of an Escrow on Age related Portfolio Composition

The starting point for Samuelson's 1989 escrow paper is the demonstration by Merton (1969) that, for a wide class of attitudes towards risk, it is optimal for individuals to invest a constant proportion of their assets in a risky asset bundle, with the rest in safe. Taken at face value, this precludes age-phasing. He then posits an alternative type of investor to that captured by the wide class of attitudes to risk referred to above – one who insists that when he retires, some minimum level of wealth must be available. To guarantee this, such an individual places a proportion of his wealth in a safe cash escrow, leaving the remaining wealth to be invested in constant proportions between risky and safe.

This pattern of investment Samuelson mistakenly interprets as leading to age-phasing. The reason that we won't, in general, see age-phasing here is linked to the superior returns, on average, of risky assets. Where risky assets outperform safe assets, the gradually accumulating non-escrowed account comes to dominate the total portfolio, and the escrow account, accumulating at the safe rate, becomes a smaller and smaller proportion of the whole. Where this occurs, we will observe the opposite of traditional age phasing: the proportion of wealth allocated to risky will increase over time. Only in a minority of cases, where risky assets perform consistently worse than safe, would we see age-phasing.

Table 1 demonstrates this for the case of three individuals with different levels of constant relative risk aversion. Each has an initial wealth of $150 000. The expected outcome of their optimal investment in risk over their working lives is for a gradual move towards risky assets, and *not* age-phasing. Samuelson's argument, that an individual with a minimum retirement wealth target leads to a pattern of greater risk taking when young than when old, is not correct. In Appendix 1, we demonstrate this formally.

TABLE 1 HERE

3. The Effect of a Saving Rule

To begin, abstract from the motivations for age phased investment associated with Bodie et al (1992). Then the Merton (1969) constant proportions portfolio rule for optimal investment strategy will hold. Nevertheless, if the client has adopted a saving rule, a financial planner may still be offering sensible advice in suggesting an age-phased strategy.

In assuming a saving rule, we are of course departing from the strict optimising framework which lies at the foundation of the Merton (1969) analysis. Nevertheless, in the context of life cycle saving, there is considerable evidence that rationality may be bounded, and that individuals appeal to habitual rules of thumb to achieve their goals. Herbert A. Simon (1955) argues that a person, faced with a complex mental task, will not attempt to strictly optimise, but will be content instead merely to "satisfice". That is, he aims to find not the best, but a good solution – one which achieves a given proximate goal. The idea that saving is supported by habit goes back at least as far as Irving Fisher (1930), and has been developed specifically in the context of long-term saving by Thaler (1994), among others.

To illustrate how a saving rule might imply age phasing as an optimal investment strategy, suppose an individual adopts the habit of saving $*k* per annum until retirement. This is an example of a simple saving rule. The value of this stream at any point in time must be taken into account in calculating the proportion of the portfolio actually in hand to be invested in safe assets. If the Merton constant proportion rule is adopted, then age-phasing of the portfolio in hand will result, because the realised portfolio will exclude the value of the safe saving stream to come.

In contrast to the age-phasing arguments of Bodie et al (1992), the age-phasing embodied here, and presented formally in Appendix 2, equation A2.2, is generated by the assumption of a firm saving habit. However, in common with the Bodie et al analysis, it explains the age-phasing advice offered by financial planners.

Table 2 offers some illustrative estimates, using the same financial basis as that in Table 1, but excluding the escrow. These have been set up so that the present value of the “safe” saving flow is $150 000, equal to the endowment of Table 1, and with the same return process.[2] In addition, we impose a borrowing constraint under which we prevent an investor borrowing against the future value of saving. Investors only invest their saving as it accrues. In Appendix 2, we set this model out formally.

The impact of the borrowing constraint is to initially force all saving into risky. This continues for some time into the working life cycle: for example, on average, a person with typical preferences towards risk will put all accumulations into risky for 16.8 years.

TABLE 2 HERE

Our analysis recognises that a saving rule of some kind through working life will be an integral feature of most people’s financial landscapes. Consistent with Samuelson, however, we continue to finesse the question of consumption variability through working life, or between working life and retirement, even though these considerations will be critical in a full formulation of the problem.[3] The points we make do not require such a sophisticated treatment for their elucidation.

4. Conclusion

Age-phasing advice may be explained by the fact that most people adopt a “safe” saving rule of some kind through their working lives, even if other assets, including human capital are risky. Its value at any point in time will count as a safe asset in an individual’s complete financial portfolio. Typically, however, financial advisers do not count this when recommending age-phased strategies. Because the value of this uncounted safe asset declines over time, age-phasing advocacy may simply embody a recommendation for a constant proportions strategy, in the presence of a borrowing constraint and a saving rule.

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Table 1

The Impact of a Minimum Retirement Wealth Target and Risk Aversion on Life Cycle Patterns of Exposure to Risk*

		Risk aversion (γ)		
		1.5 (ω = 0.75)	2.25 (ω = 0.5)	4.5 (ω = 0.25)
ESCROW TARGET: $100 000				
Implied Sinking Fund Commitment: $14 205				
Proportion in risky				
	Start	0.679	0.453	0.226
Average after	10 years	0.689	0.459	0.229
	20 years	0.699	0.466	0.231
	30 years	0.707	0.491	0.233
	40 years	0.714	0.476	0.235
Probability of Age Phased Outcome over 40 years		11.04%	8.06%	5.59%

*Assumptions: We specify the Merton (1969) model as follows – initial wealth = $150 000, a constant safe rate of return of r = 0.05, mean rate of return on the risky portfolio of α = 0.12, and volatility of the risky portfolio of σ = 0.25. For simulation purposes, an investment decision is made every quarter of a year. The Merton model gives the optimal solution as a function of the degree of the investor's risk aversion, γ, where increasing γ indicates increasing aversion to risk. This optimum occurs when investors place a certain fraction, ω, of wealth in the risky asset. Appendix 1 provides a formal treatment of Merton's model using this notation.

Table 2

Perceived Age Phasing with a fixed saving rule*

		Risk aversion (γ)		
		1.5 (ω = 0.75	2.25 (ω = 0.5)	4.5 (ω = 0.25)
Proportion in Risky				
	Start	0	0	0
Average after	10 years	0.917	0.916	0.511
	20 years	0.912	0.632	0.320
	30 years	0.801	0.533	0.269
	40 years	0.747	0.498	0.249
Average time invested 100% risky		16.8 years	9.8 years	4.7 years
Proportion of Age Phased Cases		100%	100%	100%

* Assumptions: As for Table 1; no initial wealth.

Appendix 1: Formal analysis of the impact of an escrow on age related portfolio composition.

Samuelson's 1989 escrow paper is couched in the standard mechanisms which generate constant proportion investment in risky assets over time – an individual with constant relative risk aversion, investing in either a safe asset (with constant returns *r*) or a risky asset, whose returns process is a random walk with mean α and volatility σ. As Merton (1969) showed, such an investor, consuming *c(t)* at time t, with utility given by $U(c) = c^{1-\gamma}/(1-\gamma)$ and risk aversion γ, will invest a constant proportion $\omega = (\alpha - r)/(\gamma\sigma^2)$ in the risky asset.

With the above theory as a starting point, Samuelson asks us to consider the case of an investor, at time 0, who must attain wealth of *S* at some future date, *n*. In order to do so, the individual would put $S/(1+r)^n$ in a safe cash escrow, leaving the remaining wealth, $W_0 - \left(S/(1+r)^n\right)$ to be invested between the risky and safe assets in proportions ω and $1-\omega$ respectively. If this is the case, Samuelson shows that the amount of money invested in risky at time *t* is then

$$w_t W_t = \omega\left(W_t - \frac{S}{(1+r)^{n-t}}\right) \quad , \quad 0 \le t \le n, \tag{A1.1}$$

or

$$w_t = \omega\left(1 - \frac{S}{(1+r)^{n-1} W_t}\right) \tag{A1.2}$$

Samuelson's analysis up to this point is solid. He, however, mistakenly interprets the equation above to lead to age-phasing. To observe age-phasing, we would need to see

$$w_0 > w_1 > \ldots > w_n \tag{A1.3}$$

and this will be the case where we have

$$\frac{S}{(1+r)^n W_0} < \frac{S}{(1+r)^{n-1} W_1} < \ldots < \frac{S}{W_n} . \tag{A1.4}$$

This, however, requires

$$\frac{W_{t+1}}{W_t} < 1+r \ , \ 0 \le t \le n-1, \tag{A1.5}$$

and, hence, we won't, in general, see age-phasing in this model due to the superior returns, on average, of risky assets. Only in a minority of cases, where risky assets perform consistently worse than safe, would we see age-phasing.

Appendix 2: Formal analysis of the effect of a saving rule.

Specifically, the rest-of-working-life value of the safe saving accumulation is given by

$$\pi_t = k \cdot \int_{s=t}^{n} e^{-rs} ds \tag{A2.1}$$

So the proportion in risky of the portfolio-in-hand at any point will be given by

$$w_t = \frac{\omega\left(W_t + \pi_t\right)}{W_t} = \omega\left(1 + \frac{\pi_t}{W_t}\right) \tag{A2.2}$$

In addition, we impose a borrowing constraint under which we prevent an investor borrowing against the future value of saving, π_t. Investors only invest their saving as it accrues. The effect of such a constraint has been shown to impact optimal investment in risky as follows:[4]

$$w'_t = \begin{cases} 1 & \text{for} \quad W_t \leq \omega\left(W_t + \pi_t\right) \\ w_t & \text{for} \quad W_t > \omega\left(W_t + \pi_t\right) \end{cases} \tag{A2.3}$$

where w'_t is the proportion invested in risky of the portfolio-in-hand. The impact of the borrowing constraint is to initially force all saving into risky.

1. There are many examples of this advice, which appears to be pervasive among financial planners. Jagannathan and Kocherlakota (1996) quote several sources, including *The Wall Street Journal Guide to Planning Your Financial Future* (Kenneth Morris, Alan Siegel, and Virginia Morris, 1995); Quinn (1991, p.489), investment columnist for *Newsweek*, and Malkiel (1996, p.411), all of whom advocate age phasing strategies. Canner *et al* (1997) quote the commonly cited rule of thumb that states that the stock allocation should equal 100 minus an investor's age. They note that Fidelity Investments (1994) offers a worksheet to help investors choose a portfolio allocation, which guides investors to a conservative, moderate, or aggressive portfolio based on a combination of risk preference and time horizon. Similarly, many mutual-fund companies offer "life cycle" funds in which the portfolio mix becomes more conservative as the investor ages (Vanessa O'Connell, 1995).
2. The class of attitudes towards risk we refer to are known as constant relative risk aversion, with a risk aversion parameter greater than one. Appendix 1 gives further technical details about such beliefs.
3. This requires annual saving of $8 582 in quarterly payments.
4. Stochastic dynamic programming solutions to variants of this model are put forward by Merton (1971), Richard (1975), and Bodie et al (1992). Purcal (1997) offers a numerical solution to the problem, and shows that with declining human capital, age-phasing is likely to be observed as an optimal solution to the financial investment problem.
5. Fleming and Zariphopoulou (1991), Purcal (1997).