

BIG MATH FOR LITTLE KIDS™

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Big Math for Little Kids™ is a research-based, comprehensive, planful, and joyful mathematics program being developed for preschool and kindergarten-age children. The program is designed to capitalize on what young children know, their interests and talents; connect mathematical ideas to daily experiences and other activities they enjoy; foster exploration of mathematics in risk-free environments rich with opportunities for them to bump into new ideas; and promote discussion and reflection on their discoveries. Development of the program is being funded by the National Science Foundation (Grant # ESI-9730683) for the period, 1998-2002, and is being carried out at three sites, each of which involves a university and one or more preschools, kindergartens, or daycare/extended-day centers in low or middle-income areas. The Baltimore site is directed by Dr. Robert Balfanz, Johns Hopkins University, who works with teachers and children at the Lilly A. Ross Child Development Center, a daycare center and extended-day program for preschool and kindergarten children that is located in the basement of an inner-city church. The Boston site is directed by Dr. Carole Greenes, Boston University, who works with teachers and children in the Chelsea (Massachusetts) Early Learning Center, a public school in an urban low-income area that offers preschool and kindergarten classes to all residents of the city. The New York site is directed by Dr. Herbert Ginsburg, Teachers College, Columbia University, who works with teachers and children in the preschool program in the Corpus Christi School, a parochial school that attracts low and middle-income children, and with kindergarten children in PS 207.

We began our project work by observing the mathematical behaviors of young children during free play and during their instructional programs. Thereafter, we interviewed teachers about the mathematical adventures in which they engaged their students, and talked with the children, individually and in groups of two or three, about the mathematical ideas they brought to bear during their play and teacher-led activities. We used a variety of strategies, including incomplete pictures (What's missing?) and impossible events (What's wrong? Can you fix it?), for stimulating children to tell us what they know about number, pattern, measurement, and shape. From that information, and from what we learned from research in mathematics education and educational psychology, we began to develop *Big Math for Little Kids™*.

Principles that Guided the Development of *Big Math for Little Kids™*

Builds on Children's Knowledge and Interests

From research we know that young children, as early as age three, have an informal knowledge of mathematics that is more complex and sophisticated than originally thought (Geary, 1994; Ginsburg, Pappas, & Seo, 1999 <<Carol, you switch between “1999” and “in press” (see references); which do you want?>>; Nunes & Bryant, 1994); that young children can learn more interesting and more substantial mathematics than is introduced in most preschool and kindergartens programs (Aubrey, 1993; Kaplan, Yamamoto, & Ginsburg, 1989); and that young children are ready and eager to learn *big* mathematical ideas (Greenes, 1999).

Recent research by Ginsburg et al. (1999??) on the nature of children's activities during free play has provided evidence that young children are deeply interested in activities that have connections to key mathematical ideas. In his study supported by the Spencer Foundation, Ginsburg found that 4 and 5-year olds engage in mathematical-type activities about half of the time, and that the nature of the activities and percentage of time spent with each type of activity (e.g., pattern construction occurs most often, magnitude comparison is second) do not vary by gender, socio-economic group, or ethnicity.

Integrates Mathematics into Routine Class Adventures

Daily activities in schools and daycare centers often have mathematical features which can be highlighted and set the stage for the exploration and discussion of key mathematical ideas (Ginsburg & Baron, 1993). When children line-up for recess, discussion about their positions in the line can be the basis for the development of ordinal numbers (first, second,...). When they set the table for snack time, the *matching* of cartons of milk to children and straws to cartons of milk provides opportunities for the development of the concept of one-to-one correspondence, which then sets the stage for the comparison of quantities. When children *share* a table top during drawing time, they can “get a taste” for the concept of area and of ways to equate areas.

Free play offers opportunities for involving children in mathematical explorations that are directly related to their favorite activities. For example, children who love to play with toy cars can park their cars in different garages, count and compare the numbers of cars in two (or three) garages, and identify the number of cars needed in order to park the same number of cars in each garage. The latter activity of identifying numbers of cars that can be parked so that there are the same number in each of two garages is a natural lead-in to discussion of even and odd numbers. Similarly, block play can set the stage for discussion of the attributes of three-dimensional objects, and of the symmetry in buildings.

Stories and songs, key components of preschool and kindergarten programs, are excellent vehicles for developing, concurrently, mathematical and literacy or musical (rhythm, pitch, composition) skills.

Recreational games, such as hopscotch, with squares numbered with counting numbers, can lead to the investigation of number (hopping) patterns. As children play board games in which they learn to move markers from the start to the finish position, they also gain experience with the relationship between points and spaces on a number line.

Introduces, Maintains, and Enriches Ideas in a Planful Way

Too often at the preschool and kindergarten levels, mathematics programs are limited to counting, shape identification, the identification and completion of repeating patterns, and to an introduction to some measurement comparisons. Ideas are often introduced and not revisited in a way that enhances recall. At the preschool level, in particular, the mathematics programs are often collections of activities that are only connected thematically by setting (e.g., about the pond, about the neighborhood) or by mathematical topic (e.g., patterns, number). Although several of these activities are quite interesting and rich mathematically, they are not part of a developmental sequence. For this reason, they are often used without taking into account how the key idea(s) that undergirds the activity “grows,” that is, how it should be introduced, maintained and enriched in a planful manner. By contrast, *Big Math for Little Kids*™ helps children explore big mathematical ideas in depth, and schedules that exploration over lengthy periods of time through extended activities.

Develops Complex Mathematical Ideas

From evidence gathered in our observations and interviews at the start of our project, we know that children have informal knowledge of many complex mathematical ideas, including those of symmetry, large numbers, and function, and that they are capable of and eager to explore these and other more sophisticated ideas. We also know that with the proper mentorship, they can achieve greater understanding than has been previously expected (Vygotsky, 1986). For this reason, instead of limiting our study of shape to circles, triangles, and squares, we explore the symmetries of shapes. Instead of counting to 10 or to 50, our children learn to count to their favorite “big” numbers, often in the hundreds. They use body movements to dramatize numbers, differentiating hundreds from tens and tens from ones in terms of the relative sizes of the movements (e.g., large circling arm movement represents one hundred; a forearm movement represents a ten; a finger-flicking motion represents a one.) From this dramatization they learn the difference between 245 and 425, and between 203 and 23.

Finally, from work with growing patterns, we know that children have a good sense of how to describe the functional relationship between the nature of an element in an addition pattern and its location in that pattern (e.g., a pattern in which the first flower pot has 2 flowers, the second flower pot has 3 flowers, the third flower pot has 4 flowers,...; each flower pot has one more flower than its position number), as well as to recognize and describe inverse relationships. With regard to the latter, in the *Footprints* activity in *Big Math for Little Kids*™, children investigate the relationship between size and quantity in measuring length. Children are shown a picture of two sets of footprints along a path, one set created by large feet and the other by small feet. After speculating about who made the footprints (“Old people have big feet and little kids have small feet.” “Men have big feet and girls have small feet.”) and where they were made (“In the sand at the beach.” “In wet cement.”), they have to reconcile the fact that the larger the footprint, the fewer number of footprints; the smaller the footprint, the greater number of footprints. As noted by George, age 5, “The big feet take more space so you can pack in more littles.”

Promotes Language Development and Reflection

Having children become familiar with and use the language of mathematics is a major goal of *Big Math For Little Kids*™. This language includes not only mathematical terminology and symbolism (number, pattern, equals (=), add (+), circle, triangle), but also the vocabulary of space (next to, between), of comparison (same, different), of joining (together, altogether, total), of separating (are left, take away), of cause and effect (because, since), of prediction (could happen, might happen), of duration (takes longer, is shorter), and of verification (mistake, check your answer, correct). In *Big Math for Little Kids*™, emphasis is placed on using these terms frequently and in a variety of contexts.

Learning to think about thinking is also of critical importance (Palinscar, 1986). Numerous opportunities are provided for children to talk about what they did, to consider and tell why they did it, and to use information gained from prior experience to make next decisions. In *Big Math for Little Kids*™, various types of questions are used to develop children’s reflection abilities as well as to probe their understanding of key mathematical ideas. These include questions that ask for more information or explanation (“Tell me about this.” “Can you tell me more?” “Why did you do that?”); questions that ask for reconsideration (“Would you do that again?” “How would you change what you did?”); questions that ask for speculation and hypothesis formation (“What do you think will happen if I do this?” “What do you think will happen next?” “What do you think the next block will be?”); questions that ask for the detection of errors and of ways to correct them (“What’s wrong with this picture?” “How can you fix it?”); and questions that ask for

following and interpreting another child's line of reasoning ("What was (another child) thinking?" "Why do you think (another child) decided to do that?")

In addition to developing children's abilities to reflect on what they are doing, opportunities and guidance are provided for them to enhance their discussion skills. To participate in discussions, children need to learn to listen to their peers, to follow and comment on another child's line of reasoning, and to formulate questions.

Encourages Thinking Like a Mathematician

Explorations invite children to make conjectures based on observations, find supporting and non-supporting examples, identify similarities and differences, look for patterns, make predictions, test cases, figure out what's amiss and how it might be fixed, consider different ways to think about and perhaps, solve a problem, and seek verification from an expert. As an example of ways in which *Big Math for Little Kids*™ prompts children to make conjectures, bring to bear previous knowledge, test cases, and figure out what's wrong and how to fix it, consider the activity *Frog and Bear*. In our measurement strand, children are shown a picture of an unbalanced seesaw with a frog at one end (the heavier end) and a large bear at the other end (the lighter end). After they have had some time to study the picture, children are urged to talk about what they see, identify what is "funny" or "wrong with the picture" and tell how they know, and later, describe how they would "fix it." Typical responses from the children demonstrate their understanding of *heavier/lighter* and of *balance*. "The bear is bigger. He'd be at the bottom." "The frog doesn't weigh anything. The picture is mixed up." In response to the question of how they know the picture is mixed up, one child responded, "It's logical!" After further probing, the same child responded, "Cause look." as he picked up a book in one hand and a crayon in the other and mimicked the tilt of a two-pan balance beam which the children had been using in class. In response to the teacher's question about how to fix the picture, one child responded, "Put the bear on this side (the lower side of the balance beam) and the frog there (the higher side)." When prompted to speculate about the circumstances under which the frog could be heavier than the bear, another child responded, "The bear is a balloon. So the frog is heavier! It works!"

Employs Large Group, Small Group and Individual Explorations

Activities are designed to provide opportunities for children to investigate mathematical ideas in large and small groups, and to work on other activities on their own and with members of their families. The size of the group is often determined by the nature of the mathematical ideas to be explored, the format of the activity, the equipment necessary for the exploration (e.g. water for measurement investigations, blocks for exploration of symmetry in buildings), and what children bring to the activities in terms of past experience

with the mathematics and their interests, as assessed through clinical interviews and naturalistic observations.

Providing sufficient time for the exploration of key ideas is critical, as is the amount of time devoted to each activity. Too often, in current educational practice, old activities are disbanded in favor of new ones in the name of maintaining student interest. This practice has not proved to be beneficial in promoting learning during the development of *Big Math for Little Kids*™. Children enjoy hearing the same stories repeated over long periods of time. They delight in singing the same songs for days on end. They are attentive to television programs that have a great deal of repetition, as for example, *Blues Clues*™ which airs the same program each week for five consecutive days. Children desire and need repetition of activities. They need time to grow accustomed to the format of an activity and to wrestle with important mathematical ideas. By providing children with greater experience with the same activities, children gain confidence in their abilities to play with the mathematical ideas and their recall seems to be improved.

The Curriculum

The Development

During Years I and II of the project, June, 1998 to June, 2000, activities for each of the six strands were developed at each level, preKindergarten and kindergarten, tested in at least one of the three sites, revised, retested and revised for submission to the publisher, Pearson Learning, for preparation for the Year III local field trials of the complete program. Included in each strand, along with the teacher book of sequenced activities, are an original story book, a game, and a child's activity book that highlight one or more of the key concepts in the strand. Prototypes of materials for parent and teacher preparation are being developed, as well. Based on results of the local field trials, the program will be revised for national field testing in Year IV. During the national field test, vignettes of ways in which the program can be used in different locales with varying demographics, will be developed and included in the program introduction to aid future users of the program. It should be noted, that for children whose first school experience is kindergarten, we are identifying in the Big Math program, ways in which some of the related preKindergarten activities can be used to prepare children for the kindergarten explorations.

The curriculum at each level, preKindergarten and kindergarten, is organized into six major strands.

The Strands

Number This strand focuses on the different ways in which numbers are used, including as labels (e.g., house numbers); as measurements (e.g., 3 straws long); as a way to quantify, that is, to tell how many (i.e., cardinality); and as a way to indicate position, that

is, to tell which one (i.e., ordinality). Activities develop students' abilities to count by ones into the hundreds and to do some fancy counting by 5's and 10's and 100's; identify the numbers of objects in sets, either visually or by counting, and recognize that attributes of the objects (e.g., color, size, function) do not affect their count; compare quantities identifying which is more, less, or if two or more sets have the same cardinality; identify proportional relationships involving twice or half; and estimate quantities in order to compare relative magnitudes (e.g., lots, little). Great emphasis is placed on representing numbers in different ways and to show children all representations at the same time. So, for example, beginning at the earliest stages of development, the number cards children use show the numeral, the word name, and a set of dots that correspond the number.

Shape Activities focus on recognition of two and three-dimensional shapes in varying positions and of different sizes. Students identify characteristics of the shapes (e.g., numbers of sides, angles, edges, vertices, faces; shapes of faces) and their properties (e.g., symmetry), and explore ways to partition and construct shapes from other shapes. They also develop their concept of perspective as they construct three-dimensional buildings from two-dimensional drawings of the buildings, and identify three-dimensional objects from shadows cast by their faces.

Measurement This strand focuses on developing children's understanding of the three fundamental principles of measurement, that of comparison, seriation, and iteration. When children measure the length of a straw in inches, they are comparing the length of the straw to a standard measuring tool, a ruler or tape measure. When children say that the pencil is longer than the eraser, they are comparing the length of one object to another. When children order or sequence objects by one or more attributes (e.g., height, weight, capacity, time duration), they are seriating. An important aspect of seriation is that measurement relationships are relative. For example, a pencil may be longer than an eraser and, concomitantly, shorter than a baseball bat (Greenes, 1999). For this reason, seriation is a more complex notion than comparison. When children measure a table by first laying straws, end-to-end, and counting the number of straws, they are gaining understanding of what it means to measure. When they progress to using only one straw, laying it down beginning at one end of the table, sliding it to where the second straw would lie, and so on, they are learning how to iterate measurement units to accomplish the same quantification.. In *Big Math for Little Kids*™, children gain experience with these three fundamental principles as they study length, weight, capacity, area, time, temperature, and money relationships.

Constructing and Partitioning Numbers Activities focus on developing children’s understanding of ways in which groups of objects can be put together and taken apart in preparation for the more formal exploration of addition and multiplication (the putting together operations) and subtraction and division (the taking apart operations.) Relationships between sets and their subsets, and fractional parts of regions are also investigated. Emphasis is placed on different ways to represent or model the actions, and on the appropriate language. In our work with children, we have noted that they do not have knowledge of the meaning of constructing and partitioning vocabulary including *altogether*, *in all*, *are left*, *part of a group*, and *equal groups*.

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Navigation and Spatial Concepts Emphasis in this strand is on developing students’ understanding and use of spatial vocabulary, including *up*, *down*, *above*, *below*, *in front of*, *behind*, *next to*, *between*, *to the left*, and *to the right*. Children get a feel for these spatial terms as they dramatize the story about a *Friendly Giant*, engage in treasure hunts to locate objects from clues about their positions in space relative to other objects, and follow directions to get from one place to another or navigate a maze. Children also explore ways of mapping their play space and their classroom, and bring to bear understanding of the relative locations and sizes of objects, as well as of distances between objects.

The Activities

All of the activities in the strands require many days or, in some cases, weeks for their completion. Some of the activities, those that we refer to as *past-times*, are short in duration and are repeated on a regular basis throughout the year. They are used to introduce, review or practice previously learned skills, and are often merged with the daily activities of the class. For example, line-up times are used to practice identifying who is first, second, third, last in line. Shaking the “right” hand with the teacher when invited into the classroom at the start of school or into circle time each morning helps children differentiate right from left. Performing the *Hokey Pokey* or rereading *What’s On Top ? (Big Math for Little Kids™)* helps children maintain their spatial vocabulary. Counting by ones or by tens to 100 while dramatizing the decades (e.g., the twisty teens, the hopping twenties) helps maintain counting skills.

The other activities, which constitute the bulk of the program, are sequenced in such a way as to provide children with more robust understanding of key mathematical concepts and skills in the strand and to do it in a planful and joyful way. Each activity involves

multiple tasks that are sequenced by difficulty, and that may be tackled for many days. The activities include extensions that help children apply learned skills in more complex or different settings. Teachers are provided with questions to ask that will tease out what children know about the key mathematical ideas and which will help children maneuver through the activities like mathematicians, observing, forming conjectures, testing ideas, and sharing results.

Stories in Big Math for Little Kids™

Stories and storytelling are major components of *Big Math for Little Kids™* because they facilitate children’s mathematical language development while concurrently introducing children to new mathematical ideas or providing them with opportunities to practice and demonstrate what they know. In addition to incorporating the use of outstanding commercial story books that enhance understanding of strand-related big ideas (e.g., *One Hundred Hungry Ants* for considering ways to partition a set into equal groups), we have developed stories for children to “read,” stories for children to dramatize, and pictures that prompt storytelling.

As noted earlier, story books have been developed to highlight one or more key ideas in each strand. Each of the stories is interactive, requiring children to, for example, speculate about what might happen next or what the character should do, chime in on repetitive sections, or identify shapes or numbers. After children have heard and participated in the story many times, they are each given the same story that has been partially illustrated. They complete the illustrations, tell the story to their classmates, and take their books home to retell the stories to their families.

Dramatizing stories is a vehicle we use to develop children’s understanding of concepts of number, space, and measurement by adding the feature of body movements to the language development. In these dramatizations, children participate in the story as one of the characters and mimic the character’s actions and speaking parts. For example, in one of the activities in the Navigation and Spatial Concepts strand, children pretend to be Jolly (children choose the name), the friendly giant, who while sitting in a rocking chair, rocking back and forth, high upon a hill overlooking a village in which “live boys and girls, mothers and fathers, dogs and cats and even teachers,” sees heavy rains falling and the village flooding. Jolly decides to save his friends, the boys, girls, mothers, fathers, dogs, cats and teachers. He hurries down the mountain, taking giant steps, until, of course, he gets to the flooded area, where he has to move slower through the water. When he reaches the village, Jolly lifts it up (in his giant hands, of course), high above his head, and returns to the top of the mountain, where all live “happily ever after!”

Throughout *Big Math for Little Kids*™, pictures are used to prompt story telling, as well as to assess students' understanding of key mathematical ideas. In some cases, the pictures present incomplete drawings, and the children have to talk about what's missing, and how they know. For example, in the growing patterns development, one of the Flower Pot pictures shows five flower pots, the first four of which have, from left to right, one, two, three and four flowers, respectively. The children's job is to decide what's missing, that is, the number of flowers in the fifth flower pot. When one group of our preschoolers were presented with this picture, most of the children responded with "5 flowers." Joshua, however, suggested moving the empty pot to the beginning of the line of pots. That, he pointed out, would make the pattern: 0, 1, 2, 3, 4.

In other cases, the pictures present conflicting information that the children must identify and resolve. One example of this is the *Frog and Bear* (the frog being the heavier of the two animals) from the measurement strand which was described earlier in this paper. In the shape strand, children are presented with pictures of a block tower that would be impossible to construct, and have to talk about how they know that the tower really wouldn't be able to "stay up," and consider ways of fixing it.

In still other cases, pictures are used to tell a story. The *Sequence Cards* help children build initial understanding of temporal order and its associated language (e.g., what happens first, second, next, last). Children arrange the three, four or five picture cards in a set in order in terms of time of occurrence, and then tell a story that justifies the order they have chosen. So, for example, three sequence cards that show three glasses, one empty, one half full, and one full of milk, can be ordered to tell the story, "The glass was full of milk. I drank some milk. Then I drank the rest. The glass has to be washed." Or, the glasses might be arranged to begin with the empty glass, which is then filled part way, and then to the top. Or, the glasses might be arranged to show empty first, followed by the glass being filled to the top, and concluding with the drinking of half of the milk. Some children have become interested in discovering all of the ways the three cards can be arranged and developing "reasonable" stories for each.

In Summary

Big Math for Little Kids™ is a comprehensive program for 4 and 5-year olds, that develops what children know and are capable of doing, mathematically, not what has been traditionally thought to be appropriate. The activities are carefully sequenced to introduce challenging mathematics in a such a way as to promote curiosity and excitement about learning and doing mathematics, and to help children think about how ideas from the various strands are related in order to be recalled and used at appropriate times. Throughout

the program, great emphasis is placed on the development of mathematical and mathematics-related language. During national field testing and thereafter, we hope to demonstrate that *Big Math for Little Kids*™ is an excellent means for preparing children for future success with the study of mathematics. <<Perhaps changes slightly awkward wording of final sentence.>>

References

Aubrey, C. (1993). An investigation of the mathematical knowledge and competencies which young children bring to school. British Educational Research Journal, 19, pp. 27-41.

Geary, D. (1994). Children's mathematical development: Research and practical applications. Washington, DC: American Psychological Association.

Ginsburg, H. P. & Baron, J. (1993). Cognition: Young children's construction of mathematics. In R. J. Jensen (Ed.) Research ideas for the classroom: Early childhood mathematics. Reston, VA: National Council of Teachers of Mathematics, pp. 3-21.

Ginsburg, H. P., Pappas, S., & Seo, K.H. (In press). Everyday mathematical knowledge: Asking children what is developmentally appropriate. In S. Golbeck (Ed.), Psychological perspectives on early childhood education. Mahwah, NJ: Lawrence Erlbaum Associates.

Ginsburg, H.P., Balfanz, R. and Greenes, C. (1999). Challenging mathematics for young children. In A. Costa (Ed.) Teaching for intelligence. Arlington Heights, IL: Skylight, pp. 245-258.

Greenes, C. (1999). Ready to Learn: Developing young children's mathematical powers. In J. Copley (Ed.), Mathematics in the early years. Reston, VA: National Council of Teachers of Mathematics, pp. 39-47.

Kaplan, R.G., Yamamoto, T. A., & Ginsburg, H. P. (1989). Teaching mathematics concepts. In L. B. Resnick & L. E. Klopfer (Eds.), Toward the thinking curriculum: Current cognitive research. Reston, VA: 1989 Yearbook of the Association for Supervision and Curriculum Development.

Nunes, T., & Bryant, P. E. (1996). Children doing mathematics Oxford, England: Basil Blackwell.

Palinscar, A. M. (1986). The role of dialogue in providing scaffolding instruction. Educational Psychologist, 21, pp. 73-98.

Vygotsky, L.S. (1986). Thought and language (A. Kozulin, Trans.). Cambridge, MA: The MIT Press.

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math exercises for children, worksheets for kids, math games, math quizzes, printables, online, interactive, quizzes, for kindergarten, preschool, first grade, math practice, for teachers and parents, teach your kids math, math help. These worksheets are from preschool, kindergarten, first grade to sixth graders levels of maths. If you are in search for printable materials with search ideas in mind like :math 4 kids, a math etc, then this site is for you. The following topics are covered Let us know what's wrong with this preview of BIG MATH for Little Kids by Kathryn Paulk. Problem: It's the wrong book It's the wrong edition Other. Details (if other): Cancel. Thanks for telling us about the problem. The activities, listed in this little book, are specific examples of what parents can do to help their children develop a strong foundation in math. This book would make an ideal gift for Mother's Day, Father's Day, or as a baby shower gift. It may also be used as a stocking stuffer, for the holidays, for any parent who would like to help their young child learn fundamenta The activities, listed in this little book, are specific examples of what parents can do to help their children develop a strong foundation in math. This book would make an ideal gift for Mother's Day, Father& Pre-K and K math Herbert P. Ginsburg Carole Greenes Robert Balfanz. Story books relate to units: unit 1 (number concepts): what are numbers? unit 2 (2-D and 3-D shapes): the shape of things unit 3 (patterns and logic): patterns plus unit 4 (measurement): measure up! unit 5 (number operations): working with numbers unit 6 (spatial relations): getting around. Pre-K: Each storybook issued with blackline take-home version. [v. 1] Teacher resource binder -- [v. 2] Henrietta sees numbers (unit 1) [v. 3] The trees of Mrs. McGee (unit 2) [v. 4] The table of Phinneas Fable (unit 3) [v. 5] Jenny saves t