

SEVENTH EDITION

Using Multivariate Statistics

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Preface

Some good things seem to go on forever: friendship and updating this book. It is difficult to believe that the first edition manuscript was typewritten, with real cutting and pasting. The publisher required a paper manuscript with numbered pages—that was almost our downfall. We could write a book on multivariate statistics, but we couldn't get the same number of pages (about 1200, double-spaced) twice in a row. SPSS was in release 9.0, and the other program we demonstrated was BMDP. There were a mere 11 chapters, of which 6 of them were describing techniques. Multilevel and structural equation modeling were not yet ready for prime time. Logistic regression and survival analysis were not yet popular.

Material new to this edition includes a redo of all SAS examples, with a pretty new output format and replacement of interactive analyses that are no longer available. We've also re-run the IBM SPSS examples to show the new output format. We've tried to update the references in all chapters, including only classic citations if they date prior to 2000. New work on relative importance has been incorporated in multiple regression, canonical correlation, and logistic regression analysis—complete with demonstrations. Multiple imputation procedures for dealing with missing data have been updated, and we've added a new time-series example, taking advantage of an IBM SPSS expert modeler that replaces previous tea-leaf reading aspects of the analysis.

Our goals in writing the book remain the same as in all previous editions—to present complex statistical procedures in a way that is maximally useful and accessible to researchers who are not necessarily statisticians. We strive to be short on theory but long on conceptual understanding. The statistical packages have become increasingly easy to use, making it all the more critical to make sure that they are applied with a good understanding of what they can and cannot do. But above all else—what does it all mean?

We have not changed the basic format underlying all of the technique chapters, now 14 of them. We start with an overview of the technique, followed by the types of research questions the techniques are designed to answer. We then provide the cautionary tale—what you need to worry about and how to deal with those worries. Then come the fundamental equations underlying the technique, which some readers truly enjoy working through (we know because they helpfully point out any errors and/or inconsistencies they find); but other readers discover they can skim (or skip) the section without any loss to their ability to conduct meaningful analysis of their research. The fundamental equations are in the context of a small, made-up, usually silly data set for which computer analyses are provided—usually IBM SPSS and SAS. Next, we delve into issues surrounding the technique (such as different types of the analysis, follow-up procedures to the main analysis, and effect size, if it is not amply covered elsewhere). Finally, we provide one or two full-bore analyses of an actual real-life data set together with a Results section appropriate for a journal. Data sets for these examples are available at www.pearsonhighered.com in IBM SPSS, SAS, and ASCII formats. We end each technique chapter with a comparison of features available in IBM SPSS, SAS, SYSTAT and sometimes other specialized programs. SYSTAT is a statistical package that we reluctantly had to drop a few editions ago for lack of space.

We apologize in advance for the heft of the book; it is not our intention to line the coffers of chiropractors, physical therapists, acupuncturists, and the like, but there's really just so much to say. As to our friendship, it's still going strong despite living in different cities. Art has taken the place of creating belly dance costumes for both of us, but we remain silly in outlook, although serious in our analysis of research.

The lineup of people to thank grows with each edition, far too extensive to list: students, reviewers, editors, and readers who send us corrections and point out areas of confusion. As always, we take full responsibility for remaining errors and lack of clarity.

*Barbara G. Tabachnick
Linda S. Fidell*

Multivariate Statistics. Chapter 0. 3.3.1 Quantitative variables - Interval scaled . . . We follow the convention of using n to denote the number of rows of individuals who have been observed, and p to denote the number of columns (variables). We will formalise some aspects from linear algebra that will be important in understanding multivariate analysis.