

of Nitsche. Special problems such as the effect of numerical integration, nonconforming elements, isoparametric elements, and the effect of curved boundaries are taken up in Chapter 4. Certain classes of nonlinear problems are treated in Chapter 5. A discussion of the obstacle problem, characterized by a variational inequality, is presented here together with the results of Johnson and Thomee on the minimal surface problem. Also, some results of Glowinski and Marrocco on strongly monotone operators are given. Chapter 6 is devoted to fourth-order problems, particularly the plate bending problem, and a discussion of various types of plate elements and corresponding error estimates. Included here is a delicate analysis of the nonconforming finite element called Adini's rectangle. Mixed finite elements are taken up in Chapter 7. Chapter 8 contains a summary of Ciarlet's work on finite-element methods for shells. As an epilogue, a collection of numerical results obtained from "real life" applications is provided. Then comes a detailed bibliography, a glossary of symbols, and a well-prepared index.

This is a carefully written, complete account of the finite-element method for elliptic problems written by an expert who himself has made important contributions to the subject.

While the style and arrangement is that of a textbook, the book will also serve as an excellent reference and, in terms of its broad scope, approaches a treatise on the subject. A great deal of care has been exercised by the author in selecting topics, preparing figures, and in explaining the intricacies of the method. The printing and binding are excellent and, while the book is expensive, as are all specialized works these days, it should be considered required reading for any serious student or researcher of finite-element methods.

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**Unidirectional Wave Motions.** By H. Levine. North-Holland Publishing Co., Amsterdam and New York. 1977. Pages xvi-494. Price \$69.75.

REVIEWED BY Y. H. PAO<sup>9</sup>

This book is a comprehensive treatment of mathematical solutions for wave problems governed by the hyperbolic partial differential equation of the form

$$\alpha \frac{\partial^2 y}{\partial x^2} + 2\beta \frac{\partial^2 y}{\partial x \partial t} + \gamma \frac{\partial^2 y}{\partial t^2} + \delta \frac{\partial y}{\partial x} + \mu \frac{\partial y}{\partial t} + \nu y = f(x, t)$$

where the coefficients  $\alpha, \beta, \dots$  may be constant, or functions of  $x$  and  $t$ . Initial and boundary conditions are included whenever the physics of a problem dictates. Occasionally, wave problems with higher-order time derivatives and nonlinear wave equations are discussed.

The book starts with the wave motion in a string where  $\beta = \delta = \mu = \nu = f = 0$ , and  $\alpha$  and  $\gamma$  are constant, and it ends with a discussion on random waves in a string where  $\gamma$  is a random function of  $x$ , and the dispersion relation and its role instability-gain analysis. A total of 87 topics, divided into three parts and 87 sections, are discussed in this book.

The topics are organized into sections of comparable size, marking a departure from the conventional arrangement of subject matter in chapters. Although no headings are given to each part; Part I (29 sections) deals mainly with waves in various types of strings; Part II (28 sections) discussed kinematics and kinetics of wave motion in general; and Part III (30 sections) is concerned mainly with problems in wave mechanics (Shrödinger equation).

Equal emphasis is placed on mathematical methods and physical phenomena. The former includes Fourier analysis, methods of characteristics, Green's functions, method of stationary phase and saddle points, WKBJ method, scattering matrix, integral equation formulation, and variational method; and the latter includes steady and transient motion, causality, resonance, dispersion, reflection, re-

fraction, diffraction, and interference. The scope and depth of the treatment on these topics are well beyond the level of beginning courses on mathematical analysis, or graduate courses on acoustics, optics, or seismic waves. Hence it is a good source of reference for students as well as researchers who are interested in the subject of waves.

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**A First Course in Continuum Mechanics.** By Y. C. Fung. Second Edition. 1977. Prentice-Hall, Inc., N. J. Pages 340. Price \$19.95.

REVIEWED BY J. L. SANDERS, JR.<sup>10</sup>

The student who has mastered the material in this text is indeed well prepared for further study in more advanced and more specialized courses on continuum mechanics. The author has been able to choose the essential topics, pursue them to about the right depth, and to write a text which is really of manageable proportions for a course. A certain mathematical sophistication on the part of the student is assumed but the emphasis throughout is on physical understanding and the author maintains contact with the applications. The treatment of the whole subject of material properties is, I think, particularly well done. There is a good supply of imaginative exercises for the students to chew on.

I would rather see Mohr's circle and Lamé's ellipsoids committed to limbo in favor of some room for the principle of virtual work and a bit more on the strain energy of elastic solids, but what teacher has not some commitment to his own tastes? On the whole the book is excellent.

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**Numerical Analysis of Spectral Methods: Theory and Application,** By David Gottlieb and Steven A. Orszag. CBMS-NSF Regional Conference Series in Applied Mathematics, Society of Industrial and Applied Mathematics, Philadelphia, Pa. 19103. 1977. Pages 172. Price \$12.25.

REVIEWED BY DR. G. A. SOD<sup>11</sup>

Section Headings: 1. Introduction. 2. Spectral Methods. 3. Survey of Approximation Theory. 4. Review of Convergence Theory. 5. Algebraic Stability. 6. Spectral Methods Using Fourier Series. 7. Application of Algebraic Stability Analysis. 8. Constant Coefficient Hyperbolic Equations. 9. Time Differencing. 10. Efficient Implementation of Spectral Methods. 11. Numerical Results for Hyperbolic Problems. 12. Advection—Diffusion Equation. 13. Models of Incompressible Fluid Dynamics. 14. Miscellaneous Applications of Spectral Methods. 15. Survey of Spectral Methods and Applications. Appendix 1. Properties of Chebyshev Polynomial Expansions. Appendix 2. Properties of Legendre Polynomial Expansions.

This book is aimed at the person interested in the theoretical as well as the practical application of the subject. The book is divided into two parts. The first part (Sections 1-9) contains the foundation for the subject which is laid out in great detail, including basic facts from numerical analysis and approximation theory. This part is self-contained. The second part (Sections 10-15) of the book contains terse descriptions of many practical applications of spectral methods.

In the first part of the book an abundance of examples are given clarifying many of the principles. Examples are given of problems one may encounter if spectral methods are used without much thought. Example 1.3 demonstrates a Fourier method which is stable but not

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convergent due to inconsistency. A means of correcting this (polynomial subtraction) is discussed in Section 6.

Spectral methods are extremely sensitive to the proper formulation of boundary conditions. If improper boundary conditions are mistakenly applied, some types of spectral methods may become unstable while others will remain stable but will not converge. This is discussed in some detail with examples.

The effects of time-differencing and the use of semi-implicit methods for mixed initial-boundary value problems is given a careful treatment.

The second part of the book lacks the care and detail to which the first part received. Strong claims of applicability are given without substantiation. Important applications of spectral methods are to global atmospheric flow and the shallow water equations. However, the discussion of these applications was restricted to only one sentence in the last section. The subjects included are treated in an inadequate and sometimes erroneous manner. Most details are omitted leaving the reader to consult a large number of given references. A particular example of this is Section 10.

In Section 11 there is an exceptional example. A comparison is given between a spectral method and a second and fourth-order accurate finite-difference method for a standard test problem. This comparison demonstrated that spectral methods have a definite advantage over finite-difference methods in passive advection of smooth structures as represented by the "color" problem. This demonstrates that spectral methods work extremely well in controlling dispersive errors in smooth flows.

The authors are very enthusiastic about the use of spectral methods in the treatment of discontinuities. They state that one attractive feature of spectral methods is that the region of large error in a neighborhood of a discontinuity is more localized than for finite-difference schemes. As pointed out in Section 3, a result of Gibb's phenomena is that the thickness of the overshoot in a spectral method is proportional to the shortest wave length. A completely analogous result holds for finite-difference schemes; namely, that the number of mesh points over which an overshoot is spread is independent of mesh spacing, which tends to zero linearly with the smallest wavelength on the grid. This shows that the overshoot as a result of using a spectral method is no more localized than that of using a finite-difference scheme; this contradiction is not resolved.

The authors refer to the antidiffusion method of Boris and Book (*J. Comp. Phys.*, Vol. 20, 1976, pp. 397-431) as a spectral method. This is clearly false. The antidiffusion method is a procedure which may be applied to a difference method or a Fourier transformed equation in order to remove some of the smearing caused by the use of artificial viscosity.

In summary, this book has many good qualities, primarily contained in the first part. The second part is short and somewhat inadequate. The reader should not take everything the authors say at face value, for in the second part they are trying to sell the method too hard. This in no way is an attack on spectral methods. However, the unsubstantiated claims made by the authors could hamper the development of the subject.

**Stress Waves in Nonelastic Solids.** By W. K. Nowacki. Pergamon Press, Inc., Elmsford, N. Y., 1978. Pages xii-246. Price \$25.00.

REVIEWED BY T. C. T. TING<sup>12</sup>

This 246 page book consists of 6 chapters. After presenting various constitutive assumptions for inelastic materials in Chapter 1 and some basic equations governing the continuities (and discontinuities) across a singular surface and the characteristic equations for one-dimensional wave equations in Chapter 2, the remaining 4 chapters contain solutions to specific initial and boundary-value problems. Chapter 3 deals with problems of plane longitudinal waves in semi-infinite and finite elastic/plastic bars. Waves which are spherically symmetric or cylindrically symmetric are discussed in Chapter 4. Chapter 5 contains a mixture of subjects. Plastic waves of combined stress for which the wave equations possess more than one wave speeds are presented here. Also presented in this chapter are stress waves in beams, plates, and two-dimensional stress waves. This is the only place in the book where a two-dimensional wave equation is discussed. The final chapter, the shortest of the four chapters, is devoted to thermal stress waves in elastic/plastic materials.

The book is essentially a collection of solutions to specific wave-propagation problems in elastic/plastic and elastic/viscoplastic media. Special inelastic materials such as ideally plastic, rigid unloading, and soil materials obeying Grigorian's equations are also considered. Since most solutions are excerpted from published papers, many details are omitted. Therefore the book may be difficult to read for those who are not familiar with the subject. The situation is not made easier by the fact that the sources from which the materials are drawn are sometimes not identified. The reviewer noticed that the materials presented on pages 141-148, including equations and figures, are

copied from a published paper but the authors' names of that paper are not mentioned in the text nor in the name index.

The content is in general well organized and the ideas are clearly explained. However there are few statements which are either unclear or questionable. For instances, the reviewer does not understand the distinction between a shock wave and a strong discontinuity wave stated on Page 31 even after the author emphasizes that "frequently in the American literature the term shock wave is used instead of the wave of strong discontinuity." He also does not understand the statement "the generation of a shock wave does not depend on a discontinuity of the boundary conditions but its existence is caused by the physical behaviour of the medium (i.e., its material characteristic) and the deformation of the medium." It is true that the generation of a shock wave in nonlinear media does not depend on a discontinuity of the boundary conditions but its existence is not caused entirely by the physical behavior of the medium. Consider a semi-infinite bar which occupies  $x \geq 0$  and is initially uniformly prestressed. Regardless of whether the stress-strain curve of the bar is concave or convex to the strain axis, the existence of a shock wave in the bar depends on whether the applied stress at the boundary  $x = 0$  is increasing or decreasing in time. The continuity of the boundary condition in this case is irrelevant. For linear materials, the existence of a shock wave is caused entirely by the discontinuity of the boundary conditions. In view of the fact that the book was originally written in Polish, some of the questionable statements and typographical errors may be due to the translation.

A bibliography, a name index, and a subject index are attached at the end of the book. The bibliography contains 177 references which were published up to 1970 although papers which were published as late as 1975 in Polish journals or by Polish authors are included in the bibliography. The one-page name index is quite unnecessary and may cause some confusion. Some of the authors whose papers are listed in the bibliography and are cited in the text may find their names missing in the name index.

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