Fibonacci Number, Golden Ratio and their Connection to Different Floras

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Abstract

Mathematics has its great application in expression of natural beauty of living organisms. Many floras and faunas express different mathematical formulas such as Fibonacci numbers, golden spirals etc. in different structures. In plants, golden spirals generally appear everywhere from the leaf arrangement to the patters of petals of flower, from the bracts of a pine cone to the scale of a pineapple. In the present investigation, appearance of Fibonacci number and golden spirals was studied in different plants. Out of these five plants, Fibonacci number was studied in petals of daisy and golden spiral was studied in the seeds of Sunflower, bracts of pine cone, fern –fiddle head and aloe vera plant respectively.

Keywords-*Fibonacci number, golden ratio, golden spiral, flora*

I. INTRODUCTION

Nature is truly mind- blowing connecting mathematics with different branches of science. The Fibonacci numbers has a direct link with the operational research, statistics and computational mathematics (Vajda, 2008) covering different geometric topics like Golden sections etc. The concept of Fibonacci number is widely applicable to growth of every living thing, including a cell, from a grain of wheat to a hive of bees. The universe may be chaotic and unpredictable, but it also a highly organised physical realm bound by laws of mathematics. From the ancient times, Fibonacci numbers attracted mathematicians for their unique beauty and abounded applications possessing a unique characteristic with their application in different field of scienceunrelated to mathematics.

Golden ratio or golden mean or divine proportion is a number that is approximately equal to 1.618 (Henein et al., 2011). This number is also known as irrational mathematical constant "phi" and is expressed in writing using the letter φ from the Greek Alphabet. In the simplest form, Golden ratio is the division of a line into a distinctive ratio producing an artistically pleasing fraction (Posamentier and Lehmann, 2011). Having a wide fascinating usage in art and architecture, Golden ratios was also reported to be present in different human body parts as well as plant anatomy (Livio, 2002, Hemenway, 2005, Henein et al., 2011).

In mathematics, golden ratio and the Fibonacci sequence are intimately interconnected (Dunlap, 1997). In the Fibonacci sequence each number is the sum of previous two consecutive numbers. The ratio of the any two consecutive numbers of Fibonacci series reflects the approximate value of Golden ratio. Relationship between this Fibonacci series and Golden ratio is well expressed in different floral and faunal anatomy as well as their morphology. It was extensively elaborated in previous literature that in plant leaves, branches, seeds along with petals etc. was beautifully arranged in spiral reflecting the hidden beauty of Golden ration in it (Mitchison, 1977).

Thus, present investigation has been aimed to study the fascinating expression of Fibonacci numbers and golden ratio in plant structure considering some plant as study samples.

II. FIBONACCI NUMBERS AND GOLDEN RATIO

Leonardo Fibonacci (1180- 1250) gave the general rule of formulation of Fibonacci numbers as. This celebrated sequence of integer is

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89………………………………………………..

It is to be noted those successive terms of the sequence is relatively prime.

The Golden ratio(φ) is an irrational number with several curious properties. It can be defined as that number, which is equal to its own reciprocal plus 1,ie,

$$\varphi = \frac{1}{\varphi} + 1$$

Multiplying both sides of this equation by golden ratio, we derive the interesting property that the square of the Golden ratio is equal to the simple number itself plus one,ie,

$$\varphi^2 = \varphi + 1$$

$$\Rightarrow \varphi^2 - \varphi - 1 = 0$$

The solution of this quadratic equation givesφ= 1.618033989 or φ=-0.618033989.

The first value of φ (i.e.φ= 1.618) is usually regarded as the Golden ratio. The Golden ratio is an irrational number, but not a transcended one (like π), since it is the solution to a polynomial equation.

In geometry, a Golden spiral(fig 1a & 1b) is a logarithmic spiral whose growth factor is φ, i.e. the golden ratio. A golden spiral gets wider by a factor ϕ for every quarter turn it makes. A Golden spiral with initial radius has the following polar equation

$$r = \varphi^{\theta\frac{2}{\pi}}$$

Where r is the distance from the origin (or, pole) and θ is the angle (in radians from the horizontal axis).

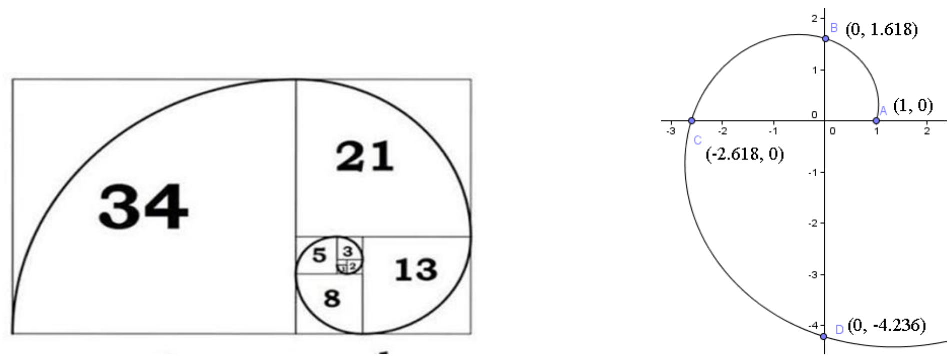

Fig. 1a & 1b Golden Spiral (Source-Google)

III. GOLDEN SPIRALS IN DIFFERENT FLORA

The spiral patterns of leaves, bracts or petals of different plants are a familiar mathematical curiosity of nature. If we closely observe and count the spirals with naked eye on the head of different flowers or a pine cone, we will able to observe in terms of a series (1, 1, 2, 3, 5, 8……….) i.e. Fibonacci series. Different plants follow this series in their phyllotaxis, petal arrangement etc. in nature. For example, lilies have 3 petals, a buttercup has 5, and marigolds bears 13, asters have 21 while most daisies have 13, 21 or 34.

Keeping these points in mind, we try to focus the golden ratio in different floras through the present investigation.

A. Sunflower

In sunflower, the construction of individual florets on the capitulum forms spirals on it by following the Fibonacci series (Mathai and Devis, 1974). In the central part, sunflower petals present in highly compressed mode forming a flat disc which expands by differentiating new petals replacing old one. The pattern of florets of sunflower (fig-2a) contains many spirals. If we count the spirals in a consistent manner, we always find a series of Fibonacci numbers. Sunflowers grow patterns of petals in a golden spiral shape (fig-2b). The spiral happens naturally because its new cell is form after a turn.

Fig. 2a Head of Sunflower

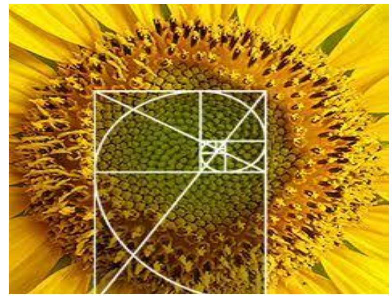

Fig.2b Expression of Golden spiral in head of Sunflower

B. Daisy

The clockwise and anticlockwise arrangement of daisy petals clearly reflects the Fibonacci series (Dixon, 1981). The petal count of field daisies is usually either thirteen, twenty one or thirty four petals, all are consecutive Fibonacci numbers. The seed head also consist of the above mentioned Golden spirals. In fig-3 the Daisy has 21 peatals.

Fig. 3 Daisy with 21 petals

C. Fern- Fiddle heads

The spiral form of a fiddle head(fig-4a) proudly displays a golden spiral.(fig-4b)

Fig. 4a Fern- Fiddle head

Fig. 4b Golden Spiral in Fern- Fiddle head

D. Aloe Vera

Many cactuses including Aloe Vera(fig-5a)lie in fairly well defined spirals(fig-5b). The numbers of scales in this spiral turn out in the Fibonacci sequence.

Fig. 5a Aloe Vera Plant

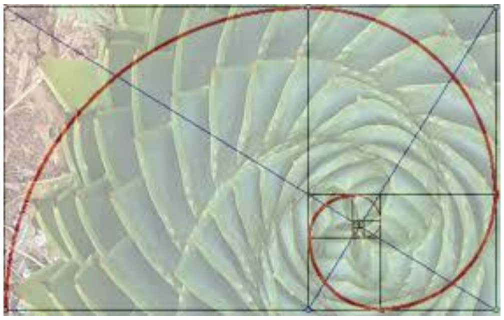

Fig. 5b Presentation of Golden spiral in Aloe Vera Plant

E. Pine cone

All pine cones grow spirally starting from the base to the top following the round pathway. Both clockwise and anticlockwise development of spiral has been documented in case of pine tree. The pine cone(fig-6a) is one of the best examples of Golden spiral, where we can easily predict eight clockwise spirals and thirteen counter clockwise spiral which is shown in the following figure(fig-6b). Counting the spirals in both directions, the resulting numbers are usually two Fibonacci numbers (0, 1, 1, 2, 3, 5, 8 …).

Fig. 6a Pine Cone

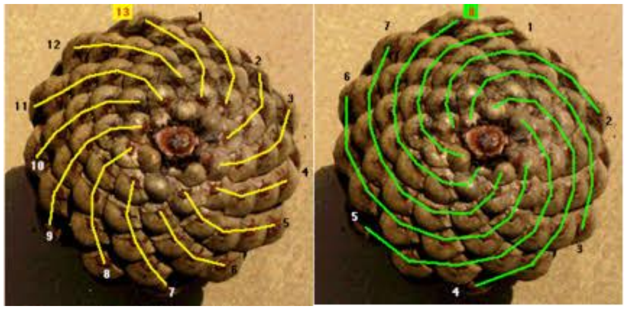

Fig. 6b Pine Cone showing 8 clockwise spirals and 13 counter clockwise spirals respectively

IV. CONCLUSION

The Fibonacci numbers are applicable to growth of every living being, including a cell. The beautiful connection between Fibonacci numbers and Golden ratios are reflected everywhere in nature, including both floral and faunal diversity. This type of hidden beauty of nature only observed through these beautiful branches of mathematics.Thus mathematics interconnected different branches of science dealing with living beings.

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ratios of successive numbers. The Fibonacci numbers play a significant role in nature and in art and architecture. When you construct a set of rectangles they both expressed movement by incorporating the golden rectangle into their paintings. The golden ratio expresses movement because it keeps on by using the golden ratio because it is pleasing to the eye. To express the Fibonacci Sequence in art Soâ€¦ x = 1+1/x (9).