

# Ludwig Prandtl's Boundary Layer

**In 1904 a little-known physicist revolutionized fluid dynamics with his notion that the effects of friction are experienced only very near an object moving through a fluid.**

## John D. Anderson Jr.

During the week of 8 August 1904, a small group of mathematicians and scientists gathered in picturesque Heidelberg, Germany, known for its baroque architecture, cobblestone streets, and castle ruins that looked as if they were still protecting the old city. Home to Germany's oldest university, which was founded in 1386, Heidelberg was a natural venue for the Third International Mathematics Congress.

One of the presenters at the congress was Ludwig Prandtl, a 29-year-old professor at the Technische Hochschule (equivalent to a US technical university) in Hanover. Prandtl's presentation was only 10 minutes long, but that was all the time needed to describe a new concept that would revolutionize the understanding and analysis of fluid dynamics. His presentation, and the subsequent paper that was published in the congress's proceedings one year later, introduced the concept of the boundary layer in a fluid flow over a surface. In 2005, concurrent with the World Year of Physics celebration of, among other things, Albert Einstein and his famous papers of 1905, we should also celebrate the 100th anniversary of Prandtl's seminal paper. The modern world of aerodynamics and fluid dynamics is still dominated by Prandtl's idea. By every right, his boundary-layer concept was worthy of the Nobel Prize. He never received it, however; some say the Nobel Committee was reluctant to award the prize for accomplishments in classical physics.

## Before Prandtl

To set the stage, let us take a quick journey back over the early development of fluid dynamics. Archimedes (287-212 BC) introduced some basic ideas in fluid statics, and Leonardo da Vinci (1452-1519) observed and drew sketches of complex flows over objects in streams. But a quantitative physical and mathematical understanding of fluid flow began-haltingly-only when Isaac Newton (1642-1727) devoted Book 11 of his *Principia Mathematica* (1687) exclusively to the examination of fluid dynamics and fluid statics. Efforts to obtain a mathematical formulation of a fluid flow took shape during the century following the publication of the *Principia* with the contribu-

tions of Daniel Bernoulli (1700-82), Jean le Rond d'Alembert (1717-83), and Leonhard Euler (1707-83)-all well-known heavy hitters in classical physics.

Of the three, Euler was the most instrumental in conceptualizing the mathematical description of a fluid flow. He described flow in terms of spatially varying three-dimensional pressure and velocity fields and modeled the flow as a continuous collection of infinitesimally small fluid elements. By applying the basic principles of mass conservation and Newton's second law, Euler obtained two coupled, nonlinear partial differential equations involving the flow fields of pressure and velocity. Although those Euler equations were an intellectual breakthrough in theoretical fluid dynamics, obtaining general solutions of them was quite another matter. Moreover, Euler did not account for the effect of friction acting on the motion of the fluid elements-that is, he ignored viscosity.

It was another hundred years before the Euler equations were modified to account for the effect of internal friction within a flow field. The resulting equations, a system of even more elaborate nonlinear partial differential equations now called the Navier-Stokes equations, were first derived by Claude-Louis Navier in 1822, and then independently derived by George Stokes in 1845. To this day, those equations are the gold standard in the mathematical description of a fluid flow, and no one has yet obtained a general analytical solution of them.

The inability to solve the Navier-Stokes equations for most practical flow problems was particularly frustrating to those investigators interested in calculating the frictional shear force on a surface immersed in a flow. This difficulty became acute at the beginning of the 20th century, with the invention of the first practical airplane by Orville and Wilbur Wright and with the subsequent need to calculate the lift and drag on airplanes. Consider the flow over the airfoil-shaped body sketched in figure 1. The fluid exerts a net force-the net aerodynamic force-on the airfoil. The figure shows the two sources of that force: the fluid pressure and the shear stress that results from friction between the surface and the flow.<sup>1</sup> The pressure and shear-stress distributions are the two hands of Nature by which she grabs hold of the airfoil and exerts a force on it.

To determine the force, aerodynamicists need to calculate both the pressure and shear-stress distributions and then integrate them over the surface of the airfoil. At the beginning of the 20th century, pressure distributions could be obtained with the help of various approximations. Pressure, however, is less problematic than shear stress, because in calculating the pressure distribution, one can assume the flow is inviscid, or frictionless. Calculating the shear-stress distribution requires the inclusion of internal friction and the consideration of viscous flow. That is, one

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needs to tackle the Navier-Stokes equations-but 100 years ago they could not be solved.<sup>2</sup>

### The boundary-layer concept

Against this backdrop, along came Prandtl and his seminal presentation at Heidelberg. The companion paper, entitled "Über Flüssigkeitsbewegung bei sehr kleiner Reibung" ("On the Motion of Fluids with Very Little Friction"), was only eight pages long, but it would prove to be one of the most important fluid-dynamics papers ever written.<sup>3</sup>

Much later, in 1928, when asked by the fluid dynamicist Sydney Goldstein why the paper was so short, Prandtl replied that he had been given only 10 minutes for his presentation, and he had been under the impression that his paper could contain only what he had time to say.<sup>4</sup>

Prandtl's paper gave the first description of the boundary-layer concept. He theorized that an effect of friction was to cause the fluid immediately adjacent to the surface to stick to the surface—in other words, he assumed the no-slip condition at the surface—and that frictional effects were experienced only in a boundary layer, a thin region near the surface. Outside the boundary layer, the flow was essentially the inviscid flow that had been studied for the previous two centuries.

The concept of the boundary layer is sketched in figure 2. In the types of flows associated with a body in flight, the boundary layer is very thin compared to the size of the body—much thinner than can be shown in a small sketch.

With the figure in mind, consider Prandtl's description of the boundary layer:<sup>3</sup>

A very satisfactory explanation of the physical process in the boundary layer [*Grenzschicht*] between a fluid and a solid body could be obtained by the hypothesis of an adhesion of the fluid to the walls, that is, by the hypothesis of a zero relative velocity between fluid and wall. If the viscosity was very small and the fluid path along the wall not too long, the fluid velocity ought to resume its normal value at a very short distance from the wall. In the thin transition layer [*Übergangsschicht*] however, the sharp changes of velocity, even with small coefficient of friction, produce marked results.

One of those marked results is illustrated in figure 2: The velocity changes enormously over a very short distance normal to the surface of a body immersed in a flow. In other words, the boundary layer is a region of very large velocity gradients. According to Newton's shear-stress law, which states that the shear stress is proportional to the velocity gradient, the local shear stress can be very large within the boundary layer. As a result, the skin-friction drag force exerted on the body is not negligible, contrary to what some earlier 19th-century investigators believed. Indeed, for slender aerodynamic shapes, most of the drag is due to skin friction.

Another marked result according to Prandtl is flow separation:<sup>3</sup>

In given cases in certain points fully determined by external conditions, the fluid flow ought to separate from the wall. That is, there ought to be a layer of fluid which, having been set in rotation by the friction on the wall, insinuates it self into the free fluid, transforming completely the motion of the latter, and there

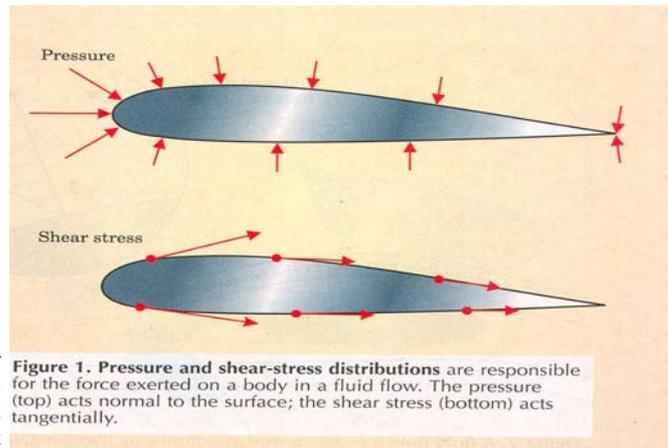


Figure 1. Pressure and shear-stress distributions are responsible for the force exerted on a body in a fluid flow. The pressure (top) acts normal to the surface; the shear stress (bottom) acts tangentially.

fore playing there the same part as the Helmholtz surfaces of discontinuity.

Prandtl was referring to the type of flow in which, as sketched in figure 3, the boundary layer separates from the surface and trails downstream. A separated flow region with some low energy flow forms in the wake behind the body, but essentially the region is dead air.

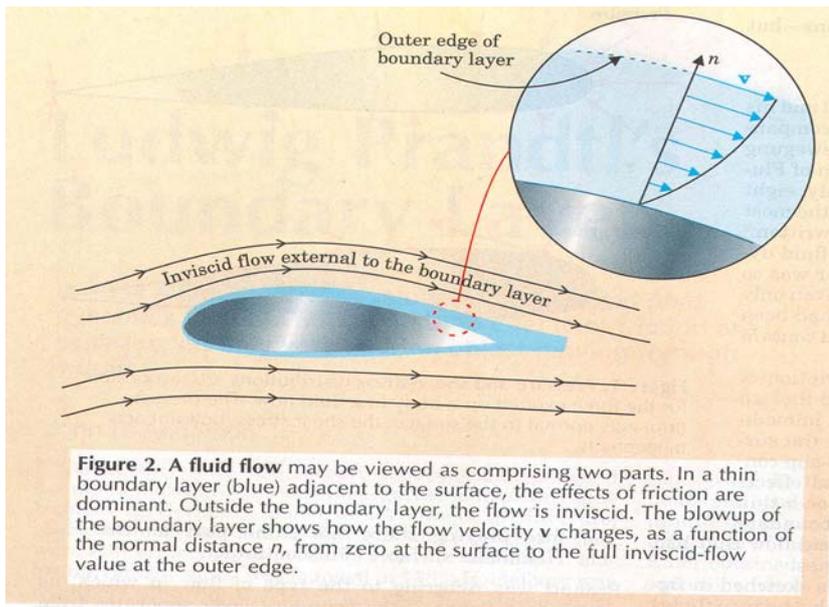
The pressure distribution over the surface of the body is radically changed once the flow separates. The altered distribution creates a pressure drag due to flow separation, that is, a large unbalanced force that acts in the direction of the free-stream flow—the drag direction. When the flow separation is extensive—that is, when the separated flow region is large—the pressure drag is usually much larger than the skin-friction drag.

The type of external inviscid flow that promotes boundary-layer separation is a flow that produces an adverse pressure gradient—in other words, an increasing pressure in the flow direction. Prandtl explained the effect as follows:<sup>3</sup>

On an increase of pressure, while the free fluid transforms part of its kinetic energy into potential energy, the transition layers instead, having lost a part of their kinetic energy (due to friction), have no longer a sufficient quantity to enable them to enter a field of higher pressure, and therefore turn aside from it.

The phenomenon described by Prandtl is illustrated in figure 3. At the separation point, the fluid elements deep inside the boundary layer have already had substantial portions of their kinetic energies dissipated by friction and so cannot work their way uphill in a region where the pressure is increasing. Hence, the velocity profile is depleted near the surface. At the separation point, it has an inflection point (see the blowup in the figure). Beyond the separation point, the boundary layer simply lifts off the surface.

In the first of the quotes above, Prandtl referred to both a transition layer and a boundary layer, and he used the terms interchangeably. In his 1905 paper, he frequently referred to a transition layer but used the term boundary layer only once. The latter term is the one that



Survived, though, mainly because of its use in subsequent papers by Prandtl's students.

### Marching toward a solution

The overall perspective set forth by Prandtl in his 1905 paper was simple and straightforward. In brief, an aerodynamic flow over a body can be divided into two regions: a thin boundary layer near the surface, where friction is dominant, and an inviscid flow external to the boundary layer, where friction is negligible. The outer inviscid flow strongly affects the boundary-layer properties; indeed, the outer flow creates the boundary conditions at the outer edge of the boundary layer and dictates the velocity profile within the layer. On the other hand, the boundary layer is so thin that it has virtually no effect on the outer inviscid flow. The exception to the no-effect rule is if the flow separates; then the outer inviscid flow is greatly modified by the presence of the separation region. As Prandtl noted in his 1905 paper:

While dealing with a flow, the latter divides into two parts interacting on each other; on one side we have the "free fluid," which [is] dealt with as if it were frictionless, according to the

Helmholtz vortex theorems, and on the other side the transition layers near the solid walls.

The motion of these layers is regulated by the free fluid, but they for their part give to the free motion its characteristic feature by the emission of vortex sheets.

With the advent of Prandtl's boundary-layer concept, it became possible to quantitatively calculate aerodynamic drag. Prandtl showed that for the boundary layer, the Navier-Stokes equations can be reduced to a simpler form, applicable only to the boundary layer. The results-called the boundary-layer equations-are similar to

Navier-Stokes in that each system consists of coupled, nonlinear partial differential equations. The major mathematical breakthrough, however, is that the boundary-layer equations exhibit a completely different mathematical behavior than the Navier-Stokes equations.

The Navier-Stokes equations have what mathematicians call elliptic behavior. That is to say, the complete flow field must be solved simultaneously, in accord with specific boundary conditions defined along the entire boundary of the flow. In contrast, the boundary-layer equations have parabolic behavior, which affords tremendous analytical and computational simplification. They can be solved step-by-step by marching downstream from where the flow encounters a body, subject to specified inflow conditions at the encounter and specified boundary conditions at the outer edge of the boundary layer. The systematic calculation yields the flow variables in the boundary layer, including the velocity gradient at the wall surface. The shear stress at the wall, hence the skin-friction drag on the surface, is obtained directly from those velocity gradients.

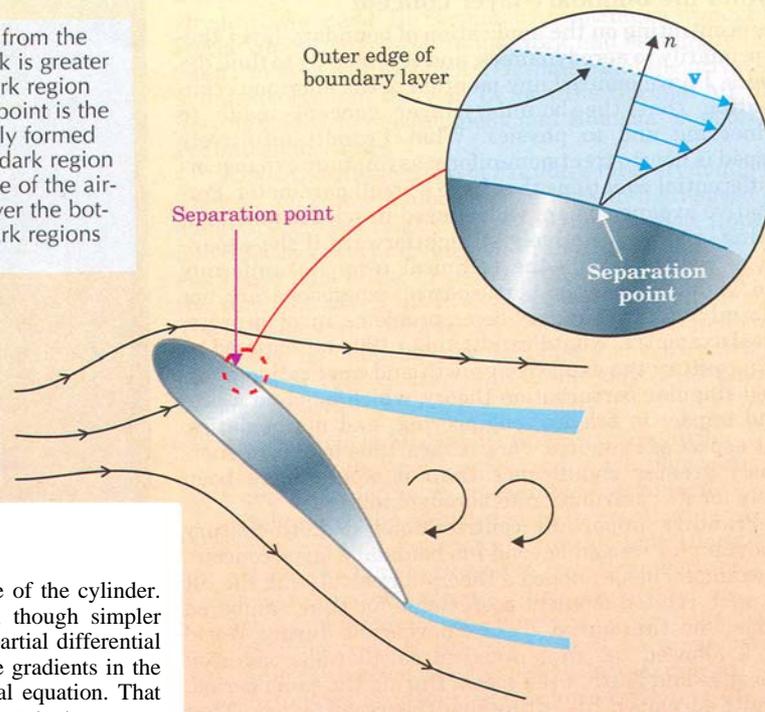
Such step-by-step solutions for boundary-layer flows began within a few years of Prandtl's 1904 presentation, carried out mainly by his students at the University of Göttingen. With those solutions, it became possible to predict with some accuracy the skin friction drag on a body, the locations of flow separation on the surface, and, given those locations, the form drag-the pressure drag due to flow separation. In his 1905 paper, short as it was, Prandtl gave the boundary-layer equations for steady 2D flow, suggested some solution approaches for those equations, made a rough calculation of friction drag on a flat plate, and discussed aspects of boundary-layer separation under the influence of an adverse pressure gradient. Those were all pioneering contributions. Goldstein was moved to state that "the paper will certainly prove to be one of the most extraordinary papers of this century, and probably of many centuries."<sup>4</sup>

### Extensions of Prandtl's work

If Prandtl had presented his paper in our electronic age of almost instant information dissemination, his boundary layer concept would quickly have spread throughout the aerodynamics community. But at the turn, of the century, information flowed much more slowly. Also, the Third International Mathematics Congress was an obscure setting for such an important contribution, and Prandtl's idea went virtually unnoticed by anybody outside of Göttingen for several years. It surfaced again in 1908 when Prandtl's student, Heinrich Blasius, published in the respected journal *Zeitschrift für Mathematik und Physik*, his paper "Boundary Layers in Fluids with Little Friction," which discussed 2D boundary-layer flows over a flat plate and a circular cylinder.<sup>5</sup>

Blasius solved the boundary-layer equations in both cases. For the flat plate, he obtained an even more accurate solution for skin-friction drag than appeared in Prandtl's original paper. For the circular cylinder, his so-

**Figure 3. The boundary layer can separate** from the top surface of an airfoil if the angle of attack is greater than the so-called stall angle. The upper dark region that trails downstream from the separation point is the remnant of the boundary layer that originally formed on the top surface of the airfoil. The lower dark region that trails downstream from the trailing edge of the airfoil is the remnant of the boundary layer over the bottom surface. When separated, these two dark regions are called shear layers, and they form the upper and lower boundaries of the separated flow region. Between the shear layers is a dead-air region. Due to the considerable flow separation illustrated here, the lift of the airfoil is dramatically reduced—the airfoil is stalled. The blowup shows the flow's velocity profile above the separation point.



lution gave the separation points on the back side of the cylinder. As noted earlier, the boundary-layer equations, though simpler than Navier-Stokes, are still coupled, nonlinear partial differential equations. However, for certain types of pressure gradients in the flow, they reduce to a single ordinary differential equation. That simplification happens, for example, for the constant pressure along a flat plate oriented parallel to the flow—that is, at a zero angle of attack. Indeed, the equation that applies to a flat plate at zero attack angle is known today as the Blasius equation.

Despite the important work by Blasius and the subsequent publication of several papers on boundary-layer theory by Prandtl's research group at Göttingen, the aerodynamics community paid little attention, especially outside of Germany. Finally in 1921, Theodore von Kármán, a former student of Prandtl's and a professor at the University of Aachen, obtained a momentum-integral equation through the simple expedient of integrating the boundary-layer equations across the boundary layer. That equation proved to be directly applicable to a large number of practical engineering problems, and with it, the boundary-layer theory finally began to receive more attention and acceptance in the technical community.

The delayed acceptance of the boundary-layer concept is illustrated by the fifth and sixth editions of Horace Lamb's classic text *Hydrodynamics*.<sup>6</sup> The fifth edition, published in 1924, devoted only one paragraph to the boundary-layer concept and described Prandtl's work as follows: "The calculations are necessarily elaborate, but the results, which are represented graphically, are interesting." In contrast, the sixth edition, published in 1932, had an entire section on boundary-layer theory and the governing equations.

Since the mid-1920s, work aimed at advancing, extending, and applying boundary-layer theory has increased exponentially. Such work has created lifetime careers for a large number of fluid dynamicists and aerodynamicists. The first serious industrial application of boundary-layer theory occurred in the late 1920s when designers began to use the theory's results to predict skin-friction drag on airships and airplanes. Prior to that time, they had been limited to using empirical data obtained primarily from wind tunnels. Such data usually were for the total drag, and the effect of skin friction was difficult to cull out. Furthermore, until the late 1920s, wind-tunnel

data were notoriously inaccurate and the designers, conservative by nature, were reluctant to hinge their designs on them. But since the late 1920s, when the accuracy and value of skin-friction formulas obtained from boundary layer theory became more appreciated, the results of the theory have become a standard tool of the airplane designer.

Physicists and engineers have written hundreds of books about various aspects of boundary-layer theory; the classic and best-known is Hermann Schlichting's *Boundary-Layer Theory*.<sup>7</sup> Schlichting was, during the early 1930s, a Prandtl student who conducted research on various aspects of flow with friction. When graduate students of fluid dynamics read Schlichting's book today, and most do, they are exposed to technical material whose roots extend back to Prandtl's 1904 presentation, communicated by an author who worked closely with Prandtl—a wonderful continuity between the past and the present for an understanding of viscous flows.

Prandtl's boundary-layer idea revolutionized how scientists conceptualized fluid dynamics. Before Prandtl, there was much confusion about the role of viscosity in a fluid flow. After Prandtl's paper, the picture was made clear; in most cases, viscosity only played a role in the thin layer of flow immediately adjacent to a surface. What a breakthrough in the analysis and understanding of a viscous flow! Before Prandtl, there was no mathematically based, quantitative means to calculate the drag due to friction on a surface immersed in a fluid flow. After Prandtl's paper, the fluid dynamicist could quantitatively calculate the skin-friction drag. Before Prandtl, there was no understanding of the physical mechanism that caused a flow to separate from a surface. After Prandtl's paper, the physics of separated flow became clear and the understanding of fluid dynamics underwent a revolutionary change.

### Beyond the boundary-layer concept

In concentrating on the application of boundary-layer theory primarily to aerodynamics, and secondarily to fluid dynamics, I have omitted any mention of the enormous contribution that the boundary-layer concept made to engineering and to physics. What Prandtl intuitively grasped is how to treat nonuniform asymptotic expansions of differential equations that have a small parameter. Perturbative expansions are widely used in science and engineering, and are relatively straightforward if the expansion is well behaved—the technical term is "uniformly valid." Often, however, perturbative expansions are not uniformly valid; boundary-layer problems involve prototypical examples. What Prandtl did in 1904 anticipated by half a century the explosive growth and application of so-called singular perturbation theory, which has had a profound impact in science, engineering, and mathematics. That aspect of Prandtl's work makes 1904 a year of enormously greater significance than it would have been purely for its contribution to aerodynamics.

Prandtl's important contributions to 20th-century aerodynamics ranged beyond his boundary-layer concept. For example, he developed a theory for calculating the lift and pitch-related moment coefficients for thin, cambered airfoils; his thin-airfoil theory, developed during World War I, allowed the first practical calculations of airfoil properties and is still used today. During the same period, Prandtl developed his lifting-line theory for wings. That theory, which also is still in common use, gave the first method for calculating how the aspect ratio of the wing affects drag, confirmed the existence of induced drag due to the presence of wingtip vortices, and provided an engineering method for the accurate calculation of that drag.

In high-speed aerodynamics, Prandtl and his student Theodor Meyer developed the first theory for calculating the properties of oblique shock and expansion waves in a supersonic flow; that work was the topic of Meyer's dissertation in 1908, four decades before the first supersonic airplane. In the 1920s Prandtl, simultaneously with Hermann Glauert in England, developed the first rule for correcting low-speed air foil lift coefficients to take into account compressibility effects at high subsonic speeds very useful for the high-speed airplanes of World War II. In 1929 Prandtl and Adolf Busemann first applied the rigorous method of characteristics (a technique for numerically solving hyperbolic partial differential equations) to design the proper shape for a supersonic nozzle. All designs for supersonic wind-tunnel nozzles and rocket engine nozzles use the same basic approach today. These are only a few of Prandtl's many contributions to fluid dynamics and to the field of mechanics in general.

#### Prandtl's early life

Prandtl was born 4 February 1875, in Freising, Bavaria.

His father, Alexander Prandtl, was a professor of surveying and engineering at the agricultural college at Weihenstephan, near Freising. Although the Prandtls had three children, two died at birth, and Ludwig grew up as an only child. His mother suffered from a protracted illness, and partly as a result of that, Prandtl became very close to his father. At an early age he became interested in his father's books on physics, machinery, and instruments. Perhaps his remarkable ability to go straight to the heart of a physical problem can be traced to his childhood environment; his father, a great lover of nature, taught him to observe natural phenomena and reflect on them.

In 1894 Prandtl began scientific studies at the Technische Hochschule in Munich, where his principal teacher



Figure 4. Ludwig Prandtl, photographed by Fr. Struckmeyer, Göttingen. (Courtesy of AIP Emilio Segrè Visual Archives, Landé Collection.)

was the well-known mechanics professor August Foppl.

Six years later he graduated from the University of Munich with a PhD; Foppl was his adviser. By that time Prandtl was alone; his father had died in 1896, and his mother in 1898.

Before 1900 Prandtl showed no interest in fluid mechanics. His PhD work at Munich had been in solid mechanics—unstable elastic equilibria in which bending and distortion acted together. He continued his interest and research in solid mechanics through most of his life, but that work was overshadowed by his contributions to the study of fluid flows. Soon after graduation from Munich, Prandtl had his first significant encounter with fluid mechanics. Having joined the Nürnberg works of the Maschinenfabrik Augsburg as an engineer, Prandtl worked in an office designing mechanical equipment for the new factory. He was assigned to redesign a suction device to collect lathe shavings. Finding no reliable information in the scientific literature on the fluid mechanics of suction, Prandtl carried out experiments to answer a few fundamental questions about the flows he had to deal with. The result was his new design for a shavings collector. He subsequently modified the apparatus with pipes of improved shapes and

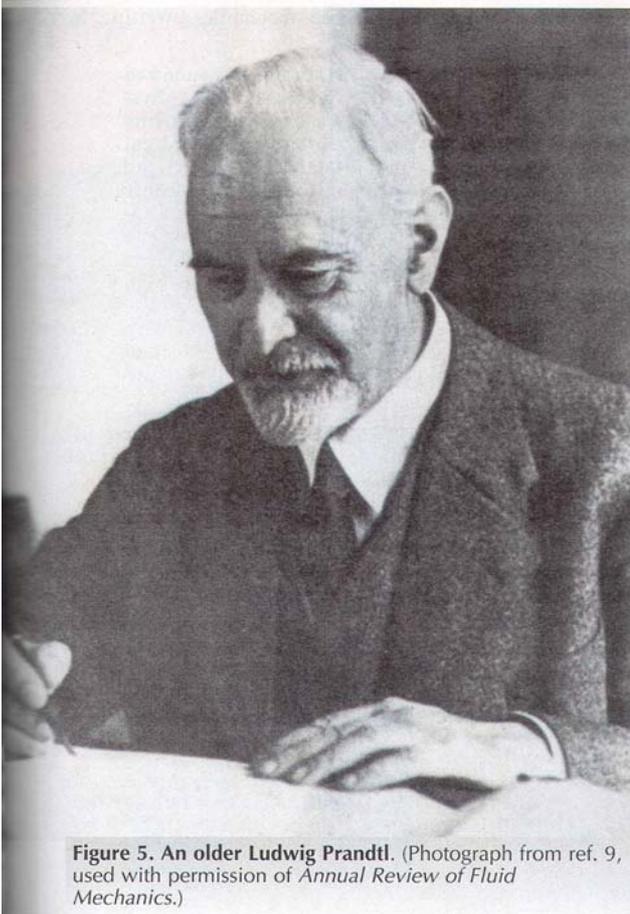


Figure 5. An older Ludwig Prandtl. (Photograph from ref. 9, used with permission of *Annual Review of Fluid Mechanics*.)

sizes, and it operated well at one-third the power consumption of the design he was assigned to improve. His contributions in fluid mechanics had begun.

A year later, in 1901, Prandtl became a professor of mechanics in the mechanical engineering department at the Technische Hochschule in Hanover, where he developed his boundary-layer theory and began work on supersonic flows through nozzles. After he delivered his famous presentation in 1904, Prandtl's star would rise meteorically. Later that year he moved to the prestigious University of Göttingen to become director of the Institute for Technical Physics. He spent the remainder of his life there and built his laboratory into the greatest aerodynamics research center of the early 20th century.

In 1909, following a tradition that seemed to prevail in German academia, Prandtl married Gertrude Foppl, the eldest daughter of his principal teacher and mentor. The marriage was a happy one, and the Prandtls had two daughters.

#### **Prandtl: The man**

In 1925, the Kaiser Wilhelm Institute for Flow Investigation was built on the grounds of the University of Göttingen. Prandtl was named as its first director in recognition of his important research achievements in mechanics. By 1930, Prandtl was recognized worldwide as the elder statesman of fluid dynamics. He continued to do research in various areas, including structural mechanics and me

teorology, but his great contributions to fluid dynamics had already been made. He remained at Göttingen throughout World War II, engrossed in his work and seemingly insulated from the politics of Nazi Germany and the privations and destruction of the war. In fact, the German Air Ministry provided new equipment and financial support for Prandtl's laboratory.

Klaus Oswatitsch, one of Prandtl's later students who went on to become famous for his work in high-speed gas dynamics, in 1987 related an interesting anecdote from the mid-1930s concerning one of Prandtl's colleagues. The story says a lot about Prandtl's professional reputation at the time and about his attitude toward the Nazi regime. The colleague was Johann Nikuradse, who was known for some landmark data on turbulent flow that were published in 1932 and 1933 and are still used today as a standard for comparison.

Nikuradse published his test results on turbulent flow through smooth and rough pipes; in order to define a special but reproducible roughness, the so-called sand grain roughness was invented. For many technical applications these two papers proved to be very important and were widely acknowledged. Unfortunately, this increased his self-esteem to such a height that he tried to replace Prandtl as director after Hitler had come to power. It was, indeed, a dangerous attack, for Nikuradse knew at least one man high up in the Nazi regime, whereas neither Prandtl nor Betz [Albert Betz, Prandtl's closest assistant] ever became party members in spite of their important positions. Luckily Prandtl was victorious. Nikuradse had to leave the Kaiser-Wilhelm-Institut and without Prandtl's guidance he never again wrote a paper worth mentioning.<sup>8</sup>

Prandtl's attitude at the end of the war was reflected in his comments to a US Army interrogation team at Göttingen in 1945: He complained about bomb damage to the roof of his house, and he asked to what extent the Americans planned to support his current and future research. Prandtl was 70 at the time and still going strong. His laboratory, however, did not fare well after the war. As related by aerodynamic engineer Irmgard Flugge-Lotz, "World War II swept over all of us [in Prandtl's laboratory]. At its end some of the research equipment was dismantled, and most of the research staff was scattered with the winds. Many are now in this country [the US] and in England [though] some have returned. The seeds sown by Prandtl have sprouted in many places and there are now many 'second growth' Göttingers who do not even know that they are."<sup>9</sup>

By all accounts Prandtl (figures 4 and 5) was a gracious man, likable and friendly, but studious and totally focused on those things that interested him. He enjoyed music and was an accomplished pianist with a preference for classical music. His students tell of Prandtl playing waltzes for dancing when they visited his home. Von Kármán mentioned in his autobiography that Prandtl bordered on being naive. Prandtl, however, was not lacking ego, as illustrated by his comment on receiving a letter announcing a new honor: "Well, they might have thought of me a bit earlier."<sup>8</sup> Although Prandtl was considered a tedious lecturer who could hardly make a statement without qualifying it, he expected his students to attend his lectures, and he attracted excellent students. Many of them—including Jakob Ackeret in Zürich, Switzerland; Busemann at various places in Germany; and von Kármán in Aachen, Germany, and later at Caltech—went on to

distinguish themselves in fluid mechanics. Writing in 1954, von Kármán commented

Prandtl, an engineer by training, was endowed with rare vision for the understanding of physical phenomena and unusual ability in putting them into relatively simple mathematical forms. His control of mathematical method and tricks was limited; many of his collaborators and followers surpassed him in solving difficult mathematical problems. But his ability to establish systems of simplified equations which expressed the essential physical relations and dropped the nonessentials was unique, I believe, even when compared with his great predecessors in the field of mechanics—men like Leonhard Euler and d'Alembert.<sup>11</sup>

Octave Chanute, a civil engineer with an intense interest in aeronautics and a close friend of the Wright brothers, made a prophetic statement in his 1894 book *Progress in Flying Machines*.<sup>12</sup> "Science," he wrote, "has been awaiting the great physicist, who, like Galileo or Newton, should bring order out of chaos in aerodynamics, and reduce its many anomalies to the rule of harmonious " law." Prandtl was that great physicist. Chanute died in 1910 without knowing of Prandtl and without knowing that in Göttingen order was being brought out of chaos in aerodynamics.

Prandtl died in 1953. He was clearly the father of modern aerodynamics and a monumental figure in fluid dynamics. The impact of his work will reverberate for as long as fluid dynamics is studied and applied.

*Some of this article has been willfully plagiarized from the author's book A History of Aerodynamics (Cambridge U. Press, 1997), which is a study of the historical evolution of our intellectual understanding of fluid dynamics in general and aerodynamics in particular.*

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Ludwig Prandtl's Boundary Layer tions of Daniel Bernoulli (1700-82), Jean le Rond d'Alembert (1717-83), and Leonhard Euler (1707-83)-all well-known heavy hitters in classical physics. Of the three, Euler was the most instrumental in conceptualizing the mathematical description of a fluid flow. He described flow in terms of spatially varying three-dimensional pressure and velocity fields and modeled the flow as a continuous collection of infinitesimally small fluid elements. Ludwig Prandtl. A Biographical Sketch, Remembrances and Documents. German original by.Â Since that time, this layer has been known as Prandtl's boundary layer. With these simplifying assumptions, the mathematical difficulties just mentioned, that show up in classical fluid mechanics of a flow with friction, could be overcome in a number of practical cases. Prandtl could prove theoretically and experimentally that the boundary layer can separate from the surface of a body immersed in a flowing fluid at suitable points, to roll up and leave the body as an isolated vortex. At the Mathematicians' Congress in Heidelberg then, Sommerfeld then heard Felix Klein saying to Prand...