

Making Mathematics Word Problems Reliable Measures of Student Mathematics Abilities

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Word problems have been used for both practicing mathematics skills and as assessments of skill mastery and application. Though students are taught the basic computational mathematics skills and even taught how to solve word problems, word problem narratives continue to be problematic for students. This study attempted to discover if the readability level at which the word problem narratives were written might be a causal factor in this ongoing difficulty with word problems. Word problems at three different levels were analyzed to ascertain their grade level readability to determine if they might prove difficult to comprehend for students at the corresponding grade levels.

Key words: word problems, mathematics skill practice, assessment, readability level.

Math word problems have been utilized since Babylonian times (Friberg, 1985) as venues of practice and evidence of mastery of learned mathematical skills. All grade levels from first grade through college utilize word problems and they are generally used in one of just a few ways. Word problems can be used as a method to practice discrete mathematical skills, such as when children are asked:

Find the planting area of a garden when the short sides of the surrounding garden fence measure 4 feet each and the long sides of the garden fence measure 8 feet each.

Word problems can also be instrumental as a demonstration of mastery of skills when students are asked to solve a complex problem and in order to succeed they have to make decisions as to the pertinent information within the problem, choose the appropriate strategies needed to help them solve the

problem, and then be able to derive the solution, such as when students are asked the following:

Flying to Kampala with a tailwind, a plane averaged 158 km/h. The plane had only half a passenger load and luggage capacity was also at half. On the return trip, the plane only averaged 112 km/h while flying back into the same wind, however it had a full passenger load and luggage capacity was at maximum. Find the speed of the wind and the speed of the plane in still air.

Word problems present unique challenges for teachers because even though teachers strive to teach the mathematical skills necessary to solve the word problem, students continue to find the narratives problematic and confusing. Though word problems are prevalent at all educational levels and appear in most cultures, yet they remain a relatively understudied aspect of mathematics learning (Reusser, 1990). Some studies have focused on identifying difficulties encountered by students as they work through a word problem and equipping them with strategies that will aid them in obtaining the solution (Griffin & Jitendra, 2009). Other studies have attempted to verify the relationship between a child's attainments in mathematics and reading while discovering the relationship between reading comprehension and strategies used by students to solve math word problems (Ansley & Forsyth, 1990; Sovik et al, 1999; Vilenius-Tuohimaa et al., 2008). The linguistic factors affecting the formation or solving of word problems have been considered as relevant factors in the study of word problems (Abedi & Lord, 2001; Lager, 2006; Reusser, 1990). Other studies centered around the analysis of methodologies used by students to solve word problems or studied the effectiveness of teaching students new methods for solving these types of problems (Bernardo, 1999; Hegarty et al., 1995; Xin et al., 2008). However, whereas the considerations in previous studies centered around the manipulation of the narrative of the word problem or the enhancement of strategies used in solving the word problems, the current study centers on the readability of the narrative itself. Since the mathematics skill/information to be studied and practiced via the word problem is delivered encased in a narrative form, the level of the language being utilized is pertinent. In an attempt to discover if the language of the word problem narrative itself might contribute to the difficulty encountered when solving word problems, this study examined the readability grade level of word problems in various textbooks currently in use in schools

in California. The readability grade level of the word problems was then compared to student proficiencies in reading and math as indicated on state assessment reports for the corresponding grade levels.

Questions Addressed in This Study

Does the readability level of word problems in current public school textbooks in California match the reading level of the students? Do the textbooks in use by students in schools today ask them to do word problems that are written at reading levels too difficult for them to comprehend thereby making the word problem a reading problem as well as a math problem?

Methodology

This study was a quasi-experimental analysis of the readability levels of word problems in mathematics textbooks in that both the choice of textbooks and the chapters analyzed were not randomly chosen, but were choices of logic and convenience. Textbooks were chosen by their accessibility in a non-random manner from districts whose socio-economic populations were similar from a common urban area. This analysis was supplemented by a summation of responses by students in an informal forum regarding personal perceptions of word problems. Mathematics textbooks were chosen from an elementary school (fourth grade), a middle school (sixth grade) and a high school (algebra) to obtain data from different levels of educational learning in order to discover patterns since the teaching of mathematics is manifested differently at these three levels of education. Three chapters in each text were chosen for analysis from the beginning, the middle, and the end of the book. They were chosen from these different areas since textbooks are designed to begin with a review of previous learning at the beginning of the text and progress to more complex material towards the end. Word problems in each of the chapters were analyzed using three different readability formulas: Flesch Reading Ease, Fog Scale Level, and the Flesch-Kincaid Grade level. These formulas indicate the ease of readability (Flesch), the word and sentence lengths (Fog), and grade level (Flesch-Kincaid) indicating the level at which a child is expected to be proficient in reading so that he easily reads and comprehends the words and their underlying message. This software was chosen due to the information it provided, its ease of use, and its ability to a wide range of input. Each word problem was typed into the software and was thereby assigned a

quantifiable readability level. These levels were then analyzed and compared and contrasted both to the expected student readability level of each grade and then to the assessed student readability level of each grade as presented by state assessment reports. For the purposes of this study, a word problem is defined as any problem requiring application of a mathematical principle in a non-mathematical setting, a setting which does not center on mathematics though it employs math. The word problem requires the student to apply mathematical computation skills in order to solve a problem designed to reflect a real-life situation.

Results

In the fourth grade text, thirty nine sections were analyzed for a total of 894 sentences. When sorted by grade level, the sections ranged in readability from .656 to 4.792, or approximately the latter part of kindergarten to the latter part of fourth grade implying that the passages should be easily readable by fluent readers in the first grade to fluent readers in the latter part of fourth grade and above. The average grade readability level for this set of word problems was 2.656. Certain sections were labeled as “challenge” word problems. When the data was disaggregated according to non-challenge (32 passages) and challenge (6 passages) word problem sections, the non-challenge word problems had a range of grade level readability level from .656 to 4.752 with an average grade level readability level of 2.616. The challenge word problem sections had a range of grade level readability from 1.994 to 4.792 with an average grade level readability level of 3.316. One-third of these challenge passages were written at the upper level fourth grade (4.6 and 4.792), yet these passages occurred in the middle of the text. Since text material is presented in a linear fashion, these passages would occur before the students were at the corresponding grade/readability level.

An analysis of aggregated passages indicated that 2.5% were written below first grade level, 21% were at the first grade level, 46% were at the second grade level, 23% were at the third grade level, and 7.5% were written at the upper fourth grade level.

Table 1

Distribution of Readability Level of Word Problems in Fourth Grade Text

Grade level	Actual distribution	% of word problems written at this level
Below 1 st grade	0.656	2.5
1 st	1.382-1.994	21
2 nd	2.037-2.911	46
3 rd	3.009-3.756	23
4 th	4.6-4.792	7.5

In the sixth grade text, thirty three sections were analyzed for a total of 708 sentences. When sorted by grade level, the sections ranged from 2.289 to 7.802 or approximately from the beginning of second grade to the latter part of 7th grade. The average grade readability level for this set of passages was 4.191. An analysis of the passages indicated that 15% were written at the second grade level, 24% were at the third grade level, 39% were at the fourth grade level, 15% were at the fifth grade level, 3% were at the mid sixth grade level, and 3% were at the upper seventh grade level.

Table 2

Distribution of Readability Level of Word Problems in Sixth Grade Text

Grade level	Actual distribution	% of word problems written at this level
2 nd	2.289-2.934	15
3 rd	3.174-3.912	24
4 th	4.086-4.789	39
5 th	5.046-5.435	15
6 th	6.5	3
7 th	7.8	3

The Mathematics Framework for California Public Schools sets forth that students should receive instruction in algebra at the high school level, with initial presentation traditionally being made in eighth, ninth or tenth grade, with mastery expected by twelfth grade. At the local high school, students are generally encouraged to take Algebra I beginning in ninth grade and are encouraged to finish Algebra II by 11th grade. The book analyzed was an Algebra II text.

Thirty one sections of the algebra text were analyzed for a total of 331 sentences. When sorted by grade level, the sections ranged from 2.954 to 12.748 or approximately from the third grade to the upper 12th grade level. The average readability level for this set of passages was 7.611. An analysis of the sections indicated that 3% were written at the second grade level, 6% at the third grade level, 3% at the fourth grade level, 10% at fifth grade level, 19% at the sixth grade level, 16% at the seventh grade level, 10% at the eighth grade level, 19% at the ninth grade level, 10% at the tenth grade level, and 3% at 12th grade level.

*Table 3***Distribution of Readability Level of Word Problems in Algebra II text**

Grade level	Actual distribution	% of word problems written at this level
2 nd	2.954	3
3 rd	3.774-3.823	6
4 th	4.368	3
5 th	5.032-5.54	10
6 th	6.384-6.952	19
7 th	7.219-7.946	16
8 th	8.336-8.911	10
9 th	9.221-9.822	19
10 th	10.213-10.767	10
12 th	12.748	3

Through the medium of an informal social network, high school students were asked: “How do you feel about math word problems? If negative, then what do you think makes them difficult? If positive, then what makes them simple for you to solve?” This informal social discussion revealed that the majority of those answering did not appreciate word problems, utilizing words or phrases such as “confusing,” “frustrating,” “weird wording,” “complicated,” several expressing a sincere hatred for them. Several students saw them as a “waste of time” not seeing any relevant need for them. A couple of the students expressed difficulty in understanding the language, stating that they were difficult reading. These statements verify the findings of other studies in which students characterized word problems as: “torture,”

“long, unnecessary. . .make no sense,” “annoying,” “time consuming,” and “senseless pieces of junk derived for the sole purpose of getting revenge on . . . students” (Smith, 1994, p.77-78). The perceptions of these students highlight the fact that the major problem identified by the students was not mathematical in nature, whereas several students expressed concerns which were language based.

Pertinent Background Data

The California Department of Education utilizes a standardized state-wide assessment system (STAR) that reports data yearly on the proficiency of students at all grades in all subjects. These assessments are standardized and administered on a yearly basis to all public school students. Information on actual student performance would enhance the discussion of the texts since it would give evidence of mastery of skill at specific grade levels. Since we are asking students to read the word problems in order to be able to solve them, data results were obtained in both language arts and mathematics. For the year 2009, in fourth grade, 66% of the students were proficient or advanced in mathematics and 61% of the students were proficient or advanced in English/Language Arts. In the sixth grade, 49% of the students were proficient or advanced in mathematics and 52% were proficient or advanced in English/Language Arts. Since the text was for Algebra II, results were gathered for students in both 10th and 11th grades. In 10th grade, 37% of students were advanced or proficient in algebra II, while 12% were advanced or proficient in 11th grade. In English/language arts, 44% of the students were advanced or proficient in 10th grade while 40% were advanced or proficient in 11th grade. At the high school level, students are given exit exams in math and language arts (CAHSEE). Of the students tested in 2008, 29% passed the math exit exam, and 30% passed the language arts exam.

Table 4

Percent of Students Proficient or Advanced on California STAR Testing Results 2009

Grade	% proficient/advanced Math	%proficient/advanced English/Language Arts
4 th	66	61

6 th	49	52
Algebra II	37	44
10 th		
Algebra II	12	40
11 th		

Discussion

Students are not performing at optimum levels in mathematics as indicated by state assessment data. Though the reports show that 66% of the students of the students in fourth grade were proficient or above in math, this leaves over one-third of the student population performing below expected mathematics levels. At the sixth grade, only 49% were proficient or above in mathematics, leaving more than half of the students falling below expectations. The gap is even greater at the high school level in algebra II where only 37% and 12% of the students were proficient or better at the 10th and 11th grades respectively. It appears that students progressively performed worse in mathematics as they moved through the grades. Since the mathematics skill/information to be studied and practiced via the word problem is delivered encased in a narrative form, the level of the language being utilized is pertinent. In the fourth grade text, most of the passages were written at a level understandable by a reader fluent at the fourth grade level, yet 39% of the students at that grade were not proficient in language arts at the fourth grade level. In the sixth grade text, most of the passages were written at a level understandable by a reader fluent at the sixth grade level, however, 48% of the students at that grade are not proficient in language arts at the sixth grade level. When you consider that only 44% and 40% of 10th and 11th grade students, respectively, are proficient in language arts, the fact that 32% of the word problems in the algebra text are written at 9th grade level or above should be a concern.

Table 5

Comparison of WP at Grade Level with Students not Proficient at That Grade

Grade	% of WP written so they are understandable at grade level	% of students <u>NOT</u> proficient in reading at this grade level
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4	92	39
6	94	48
Algebra 10	87	56
Algebra 11	87	60

The Mathematics Framework for California Public Schools identifies among its goals for students the ability to “apply mathematics to everyday life” and the “ability to use mathematical reasoning to solve mathematical problems.” (Mathematics Framework, 2006, pg. 3) The primary method used to practice skills inherent in reaching these goals is the use of word problems. However, since so many students are not proficient in reading at their current grade level, they are, in essence, being asked to solve word problems written at a language readability level which is too difficult for them to understand. Students are taught math skills at their grade level and are then asked to apply these skills by demonstrating their proficiency in deciphering a text too difficult for them to comprehend. The analysis of the textbooks indicated that the writers of the word problems assumed that children would be at least proficient readers in language at their grade level and even challenged the students with passages beyond their level of expected reading mastery. If we expect children to demonstrate mastery of mathematical skill by application through word problems, then we need to acknowledge that the word problems they are encountering in their textbooks are often too difficult for them to read, making this common vehicle for demonstration of mastery of skill acquisition an inadequate measure.

Implications for Mathematics Teaching

The teaching of mathematics, as well as the teaching reading, relies heavily on building on what has been learned before. Activation of prior knowledge and a connection to previous learning is vital for the deep learning of subject matter. Since the teaching of mathematics, as well as the teaching of reading, rely on building on what has been previously learned, on the connection with and the application of prior knowledge, it is vital that we provide sufficient practice for the new skill so that it is not just learned by rote, but learned so it can be applied in complex situations. Since we quite frequently use word problems as practice as well as assessment of mathematics skills, it seems prudent that we become more proficient in the

manner in which we structure the word problem itself. Finding the solution to a word problem should not task the reading skills of the student. The task of solving a word problem should mean that students are asked to apply math computational skills with which they demonstrate a degree of mastery to solve a word problem that is decodable, decipherable, and whose meaning is readily accessible to the reader. Simply stated, we should require students to apply only what they understand mathematically and ask them problems at a language level in which they have complete mastery.

Faced with poor student achievement in the area of mathematics at all levels and cognizant of the wide range of readability of word problems students are given to practice with, what are our options as educators? Do we cease to ask students to work through word problems, merely skipping them when they occur in our texts? Is our only option to re-write all our textbooks so that word problems are easily understandable by students at the targeted grade level, which, according to the current language assessment data, would mean that the word problems have to be decodable for readers fluent at lower grade levels? While all these texts are being rewritten, do we simply not teach word problems? Until the textbooks are changed, we need to seek other alternatives. As classroom teachers we need to work with the texts we have and further scrutinize the readability level of their word problems. We need to differentiate the manner in which we have children practice their math skills. This could be accomplished in as simple a fashion as reading the text aloud to the students so they need not have to read it by themselves, eliminating the reading factor. The problems themselves could be rewritten in the classroom in simpler language. Children could be grouped to work on their word problems allowing them to help each other with the reading of the narrative as well as the application of the math. The manner in which we assess the math skills of our students could also be differentiated, for example, students could use hands-on manipulatives to solve a problem or utilize interactive technology to aid them in seeking the solutions. We could consider using projects with multiple steps which require children to use their mathematical skills to create a product as a measure, an assessment, of their ability to apply their learned math skills.

Conclusion

Whereas other studies have attempted to investigate the interaction between reading and mathematical skills that occurs within the solving of the

word problem, this study only strove to discover the levels of readability inherent in textbooks currently in use today and to contrast the readability levels with levels of achievement in math and reading of students in corresponding grade levels. The findings lead us to make the following observations: It appears we ask students to apply mathematical skills they have not mastered; it appears we ask these students to apply these skills by deciphering language they don't comprehend. Since we assess all students at grade level, does it not appear that our methodologies and assessments are incongruent? We need to bring what we are teaching mathematically in alignment with the language of the word problems we use to have children practice and demonstrate their mastery of these skills. We should encase the mathematical problem in language that is accessible and comprehensible to the student reader.

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Beliefs about students and mathematics learning. Students are curious, active learners with individual interests, abilities and needs. They come to classrooms with varying knowledge, life experiences and backgrounds. Students learn by attaching meaning to what they do, and they need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. Through the use of manipulatives and a variety of pedagogical approaches, teachers can address the diverse learning styles, cultural backgrounds and developmental stages of students, and enhance within them the formation of sound, transferable mathematical understandings. Word problem solving in mathematics is an important aspect of learning mathematics and mathematical thinking. Unfortunately in everyday work, students exhibit difficulties solving word-problems, even when they may be skilled in performing other mathematics tasks. A student's ability to understand words in mathematics classes is related to its ability to solve word problems. Georgius (2006) found that students feel that the knowledge of the definitions of mathematical terms is significant and increases their achievements. The game required students to explain words in mathematics word problems. The words came from the daily vocabulary list (Solomon, 2009). Another activity was the presentation of words using drawings.