Mathematical Literacy

Helping Students Make Meaning in the Middle Grades

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Introduction

> Learning mathematics in the middle grades is a critical component in the education of our nation's youth. The mathematics foundation laid during these years provides students with the skills and knowledge to study higher level mathematics . . . , provides the necessary mathematical base for success in other disciplines . . . , and lays the groundwork for mathematically literate citizens.
>
> —National Research Council, *Educating Teachers of Science, Mathematics, and Technology*

This book focuses on the importance of meaning making in middle grades mathematics classrooms. We chose this focus because the middle grades (generally, grades 6–8) are an important period in the mathematical education of students. During this time, students solidify their understanding of many concepts they initially studied in elementary school, such as rational numbers and their operations, and begin more formal study of geometric and algebraic concepts. These critical mathematical turning points open doorways to later studies and careers that require mathematics prerequisites. In addition, adolescents are in a stage of development in which they particularly value peer opinions and interactions. Therefore, they need and seek opportunities to communicate with one other. We can capitalize on their need for socialization to build learning communities in which students work with their peers and the teacher to make sense of mathematics.

In this book, we share teaching practices related to communication (reading, writing, speaking, listening) that teachers can use to "establish a communication-rich classroom in which students are encouraged to share their ideas and to seek clarification until they understand" (National Research Council 2000, 270). In such classrooms, students are expected to explain their thinking, question and debate responses from their peers, and assume responsibility for their learning. Communication-rich mathematics classrooms are environments in which atmospheres of respect and trust have been developed and teachers encourage students to explain their struggles, partial understandings, and conjectures and insights about mathematics.

This book is a collaborative effort of mathematics educators and content literacy experts who have brought together their diverse classroom experiences and discipline perspectives to help clarify issues related to communication and mathematics. Many states now require teachers to have preparation in reading in the content areas, either as a requirement for initial certification or as continuing education to maintain a teaching certificate. Too often, such preparation is generic and mathematics teachers are left to determine for themselves how ideas and strategies from general content-reading courses apply to mathematics. From our experiences in teaching content-specific courses related to reading the language of mathematics, we are convinced that language-rich mathematics practices can lead to deep understanding of concepts as students work to explain their thinking to themselves and others. In the process, teachers gain valuable insights about students' emerging understandings that can inform instruction.

We have structured this book into three main sections, each with a slightly different focus. We begin each section with a brief overview that describes the format of the chapters in the section. We end each section with a brief synthesis that connects the chapters in the section.

- Chapters 1–3 in Section 1 provide a theoretical perspective on an issue related to developing mathematical literacy and meaning making. The chapters progress from general issues related to meaning making across all disciplines to broad issues about communication in mathematics to specific issues related to reading and the language of mathematics. The section introduction provides a rationale for integrating communication into the mathematics classroom from the perspective of national recommendations and describes the nature of each chapter. Chapter 1 introduces you to semiotics, a discipline that describes how individuals give and receive meaning from culturally agreed-upon sign systems, and illustrates how mathematics is such a system. Chapter 2 extends this discussion of meaning making to the role that multiple literacies (speaking, reading, writing, and listening) play in helping students make sense of mathematics. Chapter 3 follows with issues related to reading mathematical text, including issues with vocabulary and symbols, which can influence students' success with mathematics. Finally, the section concludes with a brief synthesis designed to help you connect the ideas to the mathematics classroom.
- Chapters 4–7 in Section 2 focus on literacy strategies as they might play out in a middle grades mathematics classroom. The introduction provides a rationale for the use of worthwhile mathematics tasks to engage students in communication activities to facilitate the development of mathematical literacy. The subsequent chapters provide vignettes to illustrate how communication and literacy ideas might look and sound in a classroom and how mathematical literacy can be a routine part of classroom instructional practice. Through the vignettes, we encourage you to visualize how such strategies can help with reading comprehension, vocabulary development and writing in mathematics, assessment, and the creation of a discourse community. Analysis of students' work generated with each strategy helps you understand how communication can inform instruction. The section ends with a synthesis of issues that teachers need to consider as they develop classrooms that support mathematical literacy and meaning making.
- Chapters 8–12 in Section 3 provide classroom practices that address each of the literacies (reading, writing, speaking, and listening) that we have discussed throughout this book. In addition, the first two chapters of the section

describe practices that integrate multiple literacies as students build conceptual understanding and the mathematical vocabulary necessary to discuss mathematics effectively. The summary provides opportunities to connect the practices to your own classroom.

Integrated throughout each chapter, you will find brief prompts in a callout box labeled Reflect. These prompts are designed to help you reflect on ideas presented in the chapters; we encourage you to stop and try to respond to the prompts as you read. If this book is being used as part of professional study, either as a text in a university course or as a book read by a professional learning community, the reflection prompts in each chapter provide points of discussion. At the end of each chapter, you will find two additional types of questions or prompts: Expand Your Understanding prompts are designed to help you think deeper about the issues raised in the chapter; Connect to Practice prompts provide an opportunity for you to bring the concepts you're learning into your own classroom instruction.

Whether you are a preservice, beginning, or veteran teacher, we believe this book will stimulate your thinking about enhancing communication practices in your classroom. We welcome your comments and thoughts.

CHAPTER 9

Building Meanings Through Language Development

> The investigation of the meaning of words is the beginning of education.
> —Antisthenes, c. 445–365 BC

> Learning the terms of mathematics need not be a burden. Words are a natural part of human activity; they have histories, relations to one another, and connections to the real world. Students can appreciate language and value its role in supporting communication and understanding when they are engaged in inventing, visualizing, and studying the history, uses, and connections of words.
> —Rheta N. Rubenstein, "Strategies to Support the Learning of the Language of Mathematics"

Language is the medium of teaching and learning in the classroom. Teachers and students use language to pose questions, explain their thinking, compare strategies, and provide justifications for their reasoning. We teach with language and we assess students' learning with language. We listen to students' efforts to express mathematical thinking and we read their mathematical writings. Language is central to meaning making in mathematics learning.

Like learners of any language, mathematics students face a number of challenges when learning the elements of the language system. As described in Chapter 3, the vocabulary of mathematics often creates particular difficulties for students:

- Some words in everyday language have different meanings in English and mathematics (e.g., the word *plane*).
- Some words have meaning solely in mathematics (e.g., *parallelogram*).
- Some words have comparable meanings in English and mathematics but the mathematics meaning is more precise (e.g., *similar*).
- Some words have multiple meanings within mathematics (e.g., *square*).
- Some words are learned in pairs that are commonly confused (e.g., *numerator* and *denominator*, *factor* and *multiple*, *radius* and *diameter*).

REFLECT

- What practices do you or your textbook currently use to help students identify and make sense of important mathematics vocabulary?

As teachers of mathematics, our job is to help students meet the challenges of mathematics language. How can we guide students to take ownership and gain fluency with the language of mathematics? How can we help them make sense of multisyllabic words? How can we help them compare and contrast the use of words in everyday English with the use of those words in mathematics? How can we support their understanding of meanings of terms in ways that make sense to them and are not just formal definitions? This chapter suggests practices that address these issues.

Daily Practices

There are several practices that can easily become a natural part of our daily, ongoing classroom instruction. Consider first how we often attach formal language to concepts. Mathematicians typically begin chapters in advanced mathematics books with definitions of new terms, then theorems (i.e., statements of truth) are stated and proved about these newly defined terms. This approach makes sense when the goal is to develop a deductive system in which axioms or postulates (statements assumed to be true) lead to definitions, which lead to new knowledge (theorems or statements to be proved). But, this deductive approach is not the way most mathematicians originally determined what might make a good definition. Rather, they first explored ideas connected to the topic and looked for relationships. Later they determined that a set of assumptions and definitions might make a good starting point. For example, the Euclidean geometry that students often study in school starts with assumptions of undefined terms (e.g., *point*, *line*, *plane*) and a set of postulates. Then theorems are proved based on those undefined terms and postulates. However, before Euclid organized geometric knowledge into the deductive system that we know today, ancient mathematicians explored figures and looked for connections.

As middle grades teachers, we need to approach vocabulary development in ways that make sense to learners. In middle grades instruction, it often makes more sense for learners to have terminology introduced *after* an idea has been explored rather than to start with the terminology. This enables students to focus on concepts so that language is later attached to ideas that are familiar. Think about the vignette in Chapter 1 in which Ms. Greenwood had students explore stem-and-leaf plots before they attached the name to the display.

REFLECT

- As you read the vignette, think about when language is introduced. What difficulties might arise if the teacher started with the terminology?

As another example, consider the following brief vignette.

> Mr. Bernstein wanted his students to explore ideas related to square roots. He wanted to help his students develop a solid conceptual understanding and he wanted them to have a visual model of a square root. He decided to start by having his students work on the following task with a partner:

Use base ten flats (100 square centimeters) to create another square flat whose area is 400 square centimeters.

His students had previously studied area of squares so they easily built a square with dimensions 20 centimeters by 20 centimeters. He then encouraged his students to find square flats with areas of 900 and 1600 square centimeters. Students quickly used flats to build squares with dimensions 30 centimeters and 40 centimeters on a side, respectively.

Mr. Bernstein then challenged his students to build a square with an area close to 200 square centimeters. Based on the previous squares they had built, students realized they needed to determine a number which when multiplied by itself is 200. When they couldn't build the square with base ten blocks, he suggested that they try to draw the figure on graph paper. With this small hint, his students were able to estimate the length of the side of the square needed to enclose an area of 200 square centimeters. He encouraged his students to repeat the task to find square flats with other areas, such as 300, 500, or 800 square centimeters.

When he brought the class back together, most of the students understood the underlying concept of the task (i.e., to find a number which when squared produces some other number). He then introduced the term *square root* to describe the length of the side of the square with the given area.

The vignette illustrates a useful practice in the classroom—*allow language to follow concept development*.

A second regular practice when introducing new terms is to *pronounce the word clearly, spell it, write it, use the word in a sentence*, and *engage students in these processes*. We should not assume students hear a word and connect it to the written word we intend. For example, one teacher reported hearing *Pythagorean theorem* as a child; but he thought the teacher had a lisp and meant to say *serum*. He was so distracted by his assumption of the teacher's error that he could not focus on the mathematics the teacher was addressing. In another instance, one of our colleagues used the phrase *row by column* to describe an interpretation for multiplication; her college student heard this phrase as *robot column*. Both examples reinforce our need as teachers to ensure that students hear new terms correctly, read the terms in a complete sentence, and pronounce them for themselves.

Fluency in the language of mathematics, like fluency in any spoken language, requires many opportunities to use the language, even with halting efforts. Unlike English, which students use in and out of school, students rarely use the language of mathematics outside of the mathematics classroom. We suggest two additional daily practices to support mathematics language development: make our classrooms language intense and help students find ways to use mathematics language outside of class.

REFLECT

- Recall an incident in your own learning in which not seeing a word in print or hearing it carefully pronounced led to some confusion. How was your understanding of the concept influenced?
- What are some words that your students sometimes hear or say incorrectly?
- What do you suppose students think or say to themselves when they encounter a symbol that they don't know how to pronounce?

As indicated, we need to *make the mathematics language experience within our classrooms intense*. All students need immersion into the language of mathematics. Having students work in groups provides opportunities for all students to use mathematics language. While students work in groups, teachers have a chance to monitor conversations, listen for language use (both appropriate and inappropriate), and make instructional decisions about what issues to bring to the entire class. Students can be asked to support their peers in language learning, for example, double-checking each other when they use words like *factor* and *multiple*. As students share a variety of solutions to problems and publicly question one another (with teacher support and guidance), they have many opportunities to address language issues.

Another ongoing practice to intensify mathematics language experiences is to *find ways students can use mathematical language outside of school* as part of instruction. For instance, we can assign homework that requires students to explain to someone else how and why they solved a problem as they did. When students explain a problem or process to their parents, they help their parents understand the nature of the mathematics they are studying. But more important, they have another chance to select and use appropriate mathematical terms and phrases. We can also have students listen for mathematics language in media and conversations outside of school and capture those in a section of their notebook or for a class bulletin board, perhaps providing credit for students who share such conversations with the rest of the class.

R E F L E C T

- Where do you encounter mathematics language outside of the mathematics classroom?
- Where do middle grades students encounter mathematics language that we might have them recognize?

A final ongoing practice we mention is one illustrated in the vignette in Chapter 3. We can reserve space on a bulletin board or wall for a *word* or *sentence wall* so that new terms are prominently displayed with images or definitions to remind students of their meanings. Students might make their own *personal glossaries* or *word walls* (Murray 2004; Richards 2005; Toumasis 1995) to help them clarify and own new terms and to provide a way to take the word wall home with them. (Recall the sample dictionary entries for *prime number* and *equation* from Chapter 3.)

Personal dictionaries can be constructed in several ways. Students can have one or more pieces of paper in a notebook for each letter of the alphabet. As they encounter a new word or phrase, they can write a definition in their own words, indicate the symbol used for the term if applicable, draw a diagram if appropriate, provide one or more examples and indicate why each is an example, and provide one or more non-examples and indicate why each is a non-example. Students might write entries on index cards with a hole punched in the upper corner so they can be hooked on a ring. Murray (2004) has students write words on a folder that has been partitioned into sections for each letter of the alphabet. In addition to the entries for their personal dictionaries, students can write

R E F L E C T

- What would you expect to see in a personal glossary for the term *factor*?
- What do you perceive to be the benefits of having students keep a personal glossary?

entries on large newsprint pages that can be posted on a word or sentence wall and referenced throughout the year.

Word Origins, Prefixes, and Suffixes

Technical words, such as *hypotenuse*, *perpendicular*, *polynomial*, or *isosceles*, are found only in mathematics (i.e., they are discipline specific) and seem foreign to many students. Particularly for terms that are not everyday English words, the origins or etymological roots of a word may assist students to make sense of the meaning. When they research word origins, students may find connections to words they know that help them make sense of the new term. For example, *hypotenuse* comes from the Greek words *hypo-* meaning "under" and *teinein* meaning "to stretch." In a right triangle, the hypotenuse (see Figure 9.1) is the side that "stretches" from one leg to the other "under" the right angle (assuming you draw it that way!).

Another example to illustrate the benefit of word origins is *perpendicular,* as in lines making a right angle (square corner). *Perpendicular* comes from a root meaning "plumb line," a line hanging with a weight at one end; *per* means "through" and *pend* means "to hang." When a plumb line hangs down, it creates a line perpendicular to the earth (see Figure 9.2). Students can be reminded of related English words with *pend* as one of their roots. A *pendant* hangs on one's neck. A *dependent* is a child who "hangs" onto a parent or guardian for support.

Figure 9.3 contains samples of word origins across content strands that indicate the potential of word origins to help middle grades students make sense of vocabulary.

In general, making links to related English words helps build meaning and enhances overall language development. Steven Schwartzman's book, *The Words of Mathematics*

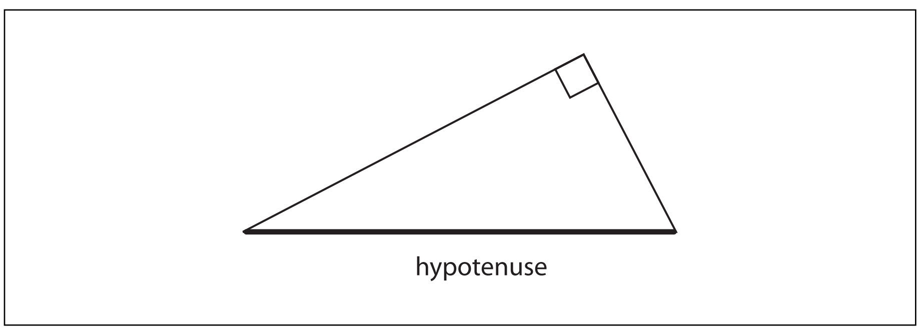


FIG. 9.1
A right triangle drawn to illustrate the meaning of hypotenuse

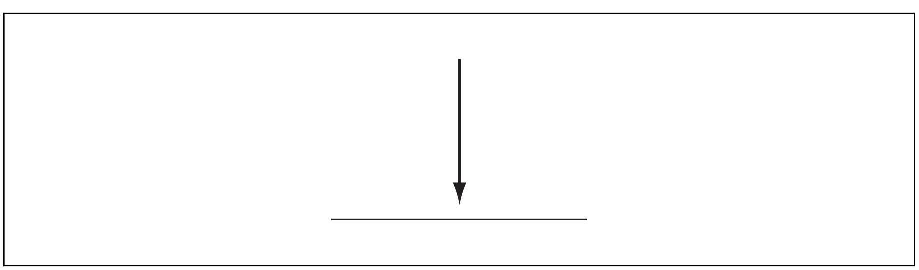


FIG. 9.2
A drawing to illustrate the meaning of perpendicular

Term and Word Origin Root	Other Related Words
Percent • *per:* for each • *cent:* 100	century centennial centipede centigram
circumference • *circum:* around or bend around • *ferre:* to bring or carry	circumnavigate circumlocution circumspect circumscribe circumvent
acute • *acute:* sharp or pointed	acute vision acupuncture acrobat
coordinate • *co:* together with • *ordinate:* a straight row	cooperate cooperative coincide
graph • *graphein:* to write, to scratch	telegraph autograph graphology graphics
reciprocal • *re:* back • *pro:* forward	reciprocity reciprocate

FIG. 9.3
Sample word origins for mathematical terms (adapted from Rubenstein 2000)

REFLECT

- Consider the origins for *reciprocal*. In what ways do the meanings of *re* and *pro* connect to how the numerator and denominator interact in a reciprocal?
- Write down as many words as you can that contain the prefix, *tri-*. What common meaning do all these words share?
- The suffix *-meter* means measure. List at least five words from middle grades mathematics that have *meter* as a root or suffix.

(1994), is an excellent source for teachers to learn etymologies of mathematics terms. Alternatively, good dictionaries often provide word origins.

In addition to the roots or origins of words, there are many common prefixes and suffixes students need to know. For example, a student who knows that *bi-* means *two* has a head start on understanding the meaning of *binomial* or *bisect*. Likewise, a student who knows that *equi-* means *equal* should be able to suggest possible meanings for *equilateral*, *equiangular*, or *equivalent*.

When teachers appropriately integrate word roots, prefixes, or suffixes into their instruction, they help students make sense of the meanings of words. However, our goal is for students to become independent word sleuths themselves. We want to look for opportunities when new terms are introduced and select students (i.e., the "Word Sleuths Team" for that unit) to research the origin of the word. They can create a poster with the word, show links to its origins (root, prefix, suffix) and, as appropriate, draw a diagram that illustrates how that meaning became attached to that word. Teachers can invite students to explain their poster to the class and to post it on the class' word wall.

Venn Diagrams

> **REFLECT**
>
> - What are some mathematics terms that also occur as everyday English words?

Language is a particular challenge to students when words are used in both everyday English and mathematics (i.e., special vocabulary), but with a more technical meaning in mathematics. For example, as indicated in Chapter 3, *similar* is one such word. In this case we want students to recognize that there is a specialized mathematical meaning and it has more details than the general everyday English meaning of *similar*. One way to help students attend to these distinctions is to invite them to create Venn diagrams to demonstrate how the same word is used in English and in mathematics. Venn diagrams may be used in conjunction with the Word Origins strategy; often origins of some root in the word provide a link that shows what the two uses have in common. Venn diagrams may be enhanced by notes or comments that detail the distinctions between the two words.

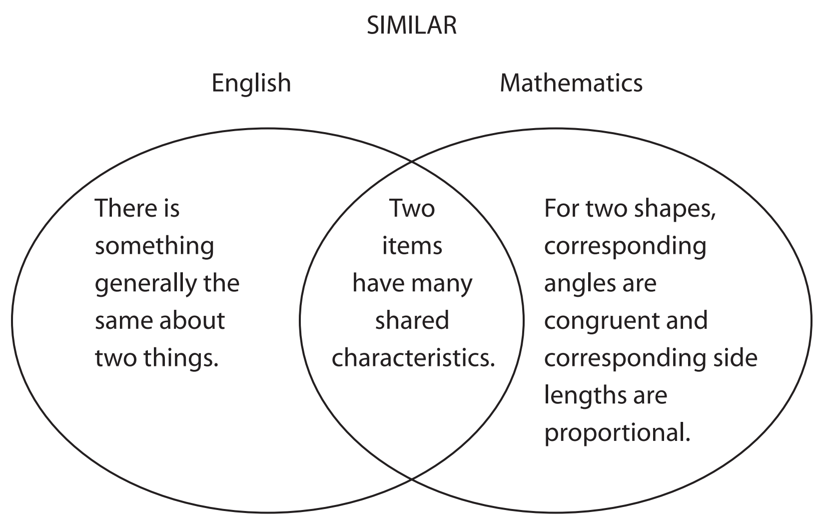
Figure 9.4 illustrates a Venn diagram for the word *similar*, showing both the distinct English and mathematical meanings as well as the sense that is shared by the two uses. A statement of distinctions between the terms accompanies the Venn diagram.

Some words, like *prime*, seem to have totally different meanings in mathematics and everyday English. But, when the origin of the word is known, commonalities emerge. Figure 9.5 demonstrates how a Venn diagram may help clarify the meanings in English and mathematics.

There are instances in which a mathematics term has different meanings in different content strands of mathematics. Venn diagrams may also be helpful here. Figures 9.6 and 9.7 for *square* and *range*, respectively, illustrate how Venn diagrams may be beneficial when words have more than one mathematics meaning.

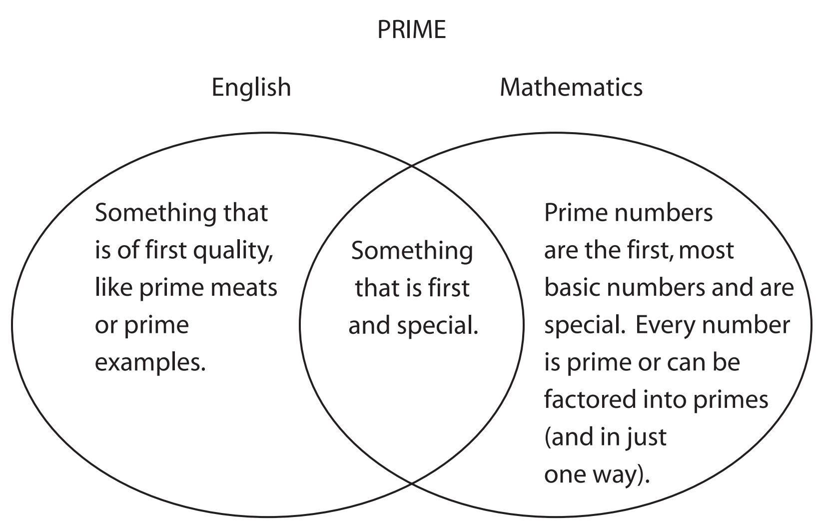
As teachers, we may provide Venn diagrams to help students appreciate the nuances of language or may develop them together as a class. However, we want students to generate their own diagrams as necessary. These diagrams may become contributions to a word or sentence wall or a personal language glossary or dictionary (as described in Chapter 3 and earlier in this chapter). The diagrams can be a source of continued attention as students gain fluency with these distinctions.

FIG. 9.4
Venn diagram for similar



Distinctions: In English, family members look similar. In mathematics, for shapes to be similar, one must be an exact enlargement or shrink of the other, like copies of the same document made on a copy machine.

FIG. 9.5
Venn diagram for prime



Distinctions: In English, *prime* is used in many situations to indicate highest or first quality. In mathematics, when any natural number is factored (represented as a multiplication) into the smallest possible natural numbers, those basics into which it eventually falls are prime numbers (i.e., 2, 3, 5, 7, 11, 13, ...). Primes are the building blocks (in terms of multiplication) of other numbers. Prime numbers have exactly two distinct factors: the number itself and 1.

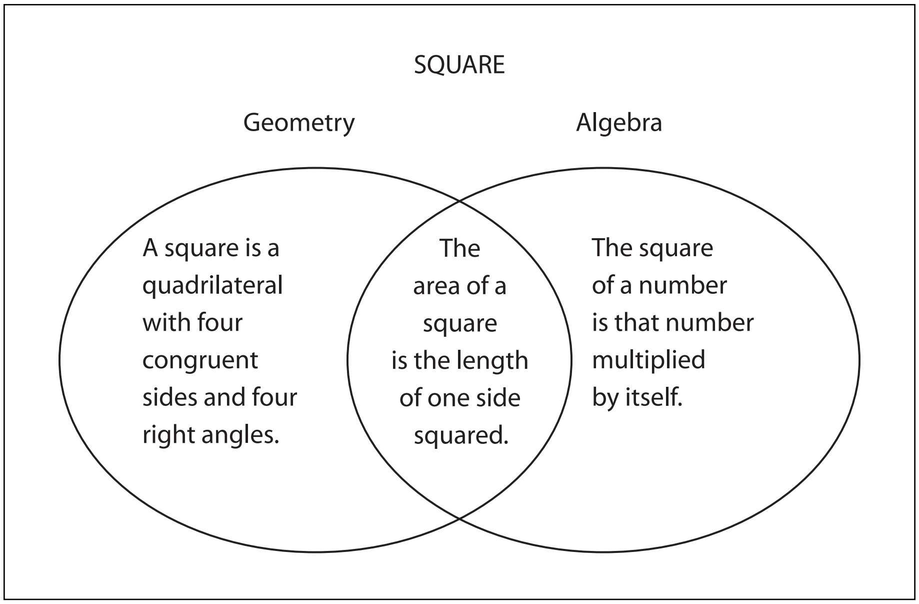


FIG. 9.6
Venn diagram for square

English language learners (ELLs) in particular may benefit from Venn diagrams. Often, ELLs first encounter a word (e.g., *similar* or *prime*) in its nontechnical setting. As teachers, we need to help students compare and contrast the everyday meaning and the less familiar mathematical meaning. In their glossaries or notes, ELL students may also include the word or words from their first language that correspond to the English and mathematical meanings.

R E F L E C T

- Modify the Venn diagram for *square* to create a diagram for *cube*.

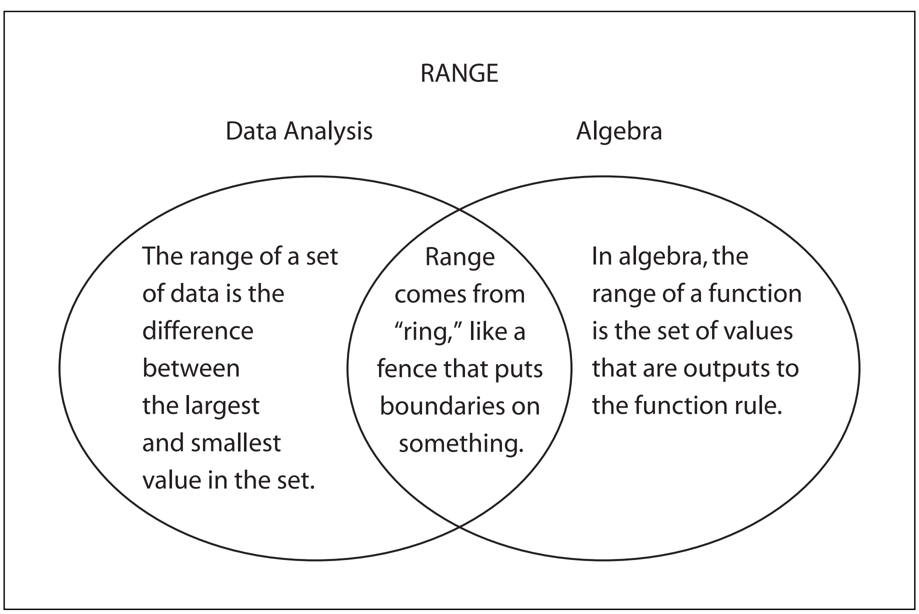


FIG. 9.7
Venn diagram for range

Distinguished Pairs

Certain pairs of words usually learned together create another language challenge for students. Examples include *numerator* and *denominator*, *radius* and *diameter*, *factor* and *multiple*, *solve* and *simplify*, *equation* and *expression*, or even *hundreds* and *hundredths*. Other pairs include homonyms or near homonyms (*pi* and *pie*, *intersect* and *intercept*), words with and without modifiers (*bisector* versus *perpendicular bisector*), or phrases with related terms (*factor list* versus *prime factorization*).

Distinguished Pairs is an activity that helps students clarify the differences by helping them find commonalities and distinctions between commonly confused word pairs. Teachers may assign different groups of students to study different difficult pairs, to focus on the distinctions between them, and to create a skit, poster, verse, Web page, or other device to dramatize what each term means and how they are different. An artifact that can serve as a reminder may be posted on the word wall.

Here are examples of distinctions students may make.

- *Numerator* and *denominator* are the two terms in a fraction. *Denominator* comes from *name* (like "nominate"). It tells the name or unit of a fraction, like fourths.

  Numerator comes from *number*. It tells the number of units there are of the denominator, like 3 in $\frac{3}{4}$ tells there are three of those things called fourths.

- *Factor* and *multiple* both have to do with multiplication. A *factor* is a number being multiplied. Factors of 12 are 1, 2, 3, 4, 6, and 12. Factors make products just like factories make products. A *multiple* is a number you get after multiplying something by some whole number. So the multiples of 12 are 12, 24, 36, 48, 60, . . . and come from 1×12, 2×12, 3×12, 4×12, and so on. A number has a multitude of multiples.

R E F L E C T

- When asked to simplify $2(x - 5)$, some students write $2x - 10$ and then continue to "solve" to obtain $x = 5$. How does confusion between *solve* and *simplify* contribute to this misconception?
- Write a sample distinction for *equation* and *expression*.

Invented Language

Despite the belief of some students that mathematics exists and has always existed independent of people and culture, mathematics, like music, visual art, writing systems, and other human communication systems, has a human history. Part of that history is the creation of words and symbols as a means to communicate the ideas of the discipline. As the word origins section suggested, terms were created or selected to relate to other words (often Greek or Latin) with meanings relevant to the concept. Oftentimes, students do not appreciate this history because they are not a part of its evolution. How might we engage students in language activities that help them understand and appreciate the thinking behind the naming of various concepts? For example, *triangle* is a shape with three (*tri-*) angles. Any one of us might have created this word, but someone else got there first! One teaching approach is to look for opportunities in which students can invent their own language.

Interestingly, the invented language strategy is easiest to use with either young or advanced (college) mathematics students. Young students may not already know formal mathematical terminology and college students may first encounter new concepts with specialized vocabulary (*group, ring, field, vector,* etc.). Middle grades students may already have heard much of the special vocabulary of mathematics. However, teachers can be alert for opportunities that encourage students to create their own informal terms.

The first vignette below illustrates one method that encourages invented language—suppress the technical terminology until students have had their own chance to suggest words. The second vignette illustrates that once the seed for invented language is planted, students may spontaneously suggest vocabulary terms.

David Whitin (1995), a mathematics educator with an interest in language, described an experience in which students explored all the rectangles that could be created with different specific numbers of square tiles (see Figure 9.8). Essentially, they were finding factorizations of the number of tiles. When students analyzed the results for many different numbers of square tiles, they concluded that some numbers made only long narrow rectangles. Other numbers made a variety of rectangles, including those that were closer to squares. The students were asked to name the two categories to distinguish the two types of numbers. Some students suggested calling the groups *sidewalk numbers* and *patio numbers,* respectively. This invented language provided an important image for the more formal terms, prime numbers and composite numbers, respectively, that were introduced later.

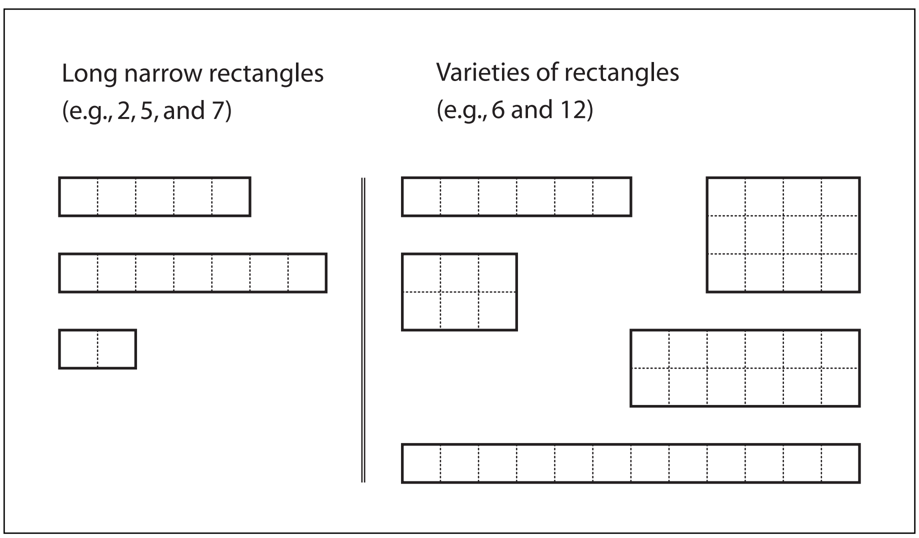


FIG. 9.8
Rectangles made with different numbers of square tiles

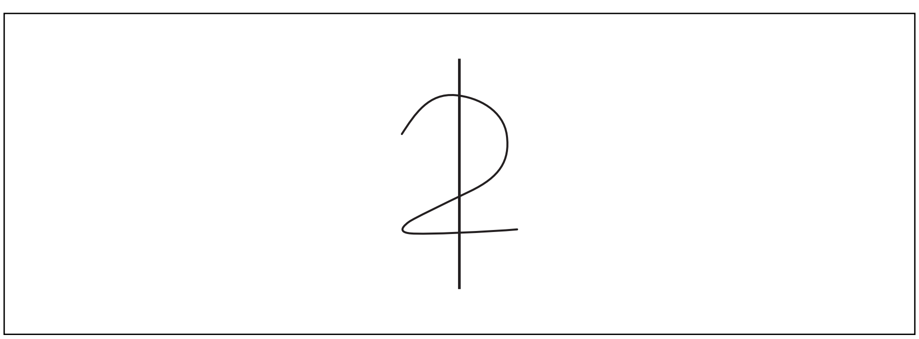
In geometry a group of high school students had learned that a midpoint divides a segment into two congruent parts. They later encountered an angle bisector that divides an angle into two congruent parts. One student wondered why they couldn't call the angle bisector a *midray*; this was a simpler term (fewer syllables) and it echoed directly the analogous concept between segments and angles. The students adopted this term. Later in the course when the students were asked to identify the points in space equidistant from two parallel planes, they chorused *midplane*. The teacher was surprised. She had been thinking instead of a plane parallel to the other two and located halfway between them. The students' spontaneous generation of another analogous term indicated their comfort with the *mid-* prefix and their easy identification of another meaningful use of it.

When we have students invent names for technical terms, at some point we need to introduce the standard term to students so they are able to communicate mathematically with individuals outside their class. Related to the sidewalk and patio numbers in the vignette, we might explain, "When mathematicians first thought about these numbers, they, too, noticed differences in the shapes of the rectangles they make. Some numbers have just one multiplication (one times the number itself), while others have many multiplications. Those with one multiplication, our 'sidewalk numbers,' are called 'prime.' Our 'patio numbers' are called 'composite.'" This would also be a good time to integrate word origins. Students might have a portion of their personal glossary in which they include both their informal, invented language and the formal, technical term.

Symbols, too, are communication tools that have human histories. As with language, the strategy of invention can be applied to symbols. We don't often think of symbolism as fair territory for invention, but it can be. For example, one of the authors of this book had a teacher who used the symbol in Figure 9.9 to mean *bisect*. (It is a 2 with a slash through it to suggest "cut in two.") It was only as an adult that she realized the teacher had invented this symbol!

Just as with vocabulary, many symbols have already been invented. But, occasionally you may find an opportunity to invite students to invent symbols. Although we can say two figures are similar and can use $\Delta ABC \sim \Delta DEF$ to denote the figures are similar, we do not have ways within diagrams to indicate that this relationship exists or that particular lengths are proportional. This lack of notation within similar figures contrasts with conventions we have to indicate that corresponding parts of figures are congruent or parallel. Perhaps a student would like to invent symbols for these needs as they arise.

FIG. 9.9
An invented symbol to mean bisect



When students invent language, they realize that all the formal terms they are expected to learn are inventions of people who created those terms because there was a need for them. Language is a human endeavor and it evolves as needs arise. This message needs to be shared when students engage in language or symbol invention. Certainly, people who create computer software regularly invent symbols (icons or buttons). Think about the use of :) at the end of an email message to mean *smile* or the use of UR on a text message on a cell phone to mean *you are*. Our students, too, can be inventors of new terms or symbols as new needs arise. A major benefit of invented terms or symbols is that students make sense of mathematics *in their own terms*.

R E F L E C T

- Middle grades students are often tech savvy. Identify at least one other symbol or letter combination (like UR) that students use when sending emails or text messages to shorten a message without losing its meaning.

Literature Connections

Middle grades students typically enjoy having a story read to them. There are many quality children's literature books that engage students and that can help them make sense of mathematics vocabulary. For instance, *The Greedy Triangle* (Burns 1994) is a story of a triangle who becomes tired of his life and believes life would be better if he had one more side and one more angle. Therefore, he visits a shapeshifter and becomes a quadrilateral. This process continues with the shape converting into a pentagon, hexagon, heptagon, and so on. The story, together with pictures and connections to real-life contexts where the shape occurs, provides an appealing introduction or review of important geometry terms and helps students identify mathematics outside the classroom.

Figure 9.10 contains a list of several children's literature books that we have found can help introduce language or concepts, either through engaging stories or through interesting pictures. When we introduce concepts or vocabulary through literature, we often start a lesson by brainstorming about the content to activate students' prior knowledge. We then read the story aloud in its entirety, pausing only to allow students to predict upcoming events. The act of predicting allows students to focus their attention on reasoning, patterns, and problem solving while adjusting their personal schemata to fit the story. To take full advantage of the motivating influence of literature, we first read the story with a focus on enjoyment; we withhold the dissection of the story's mathematics until a second pass through the story (Hunsader 2004).

Teachers and students can also create their own stories to share with the class and with younger students. We find that reading literature to the class provides a common foundation on which we can build our lesson. The content of the story helps students make sense of the mathematics in a nonthreatening manner. The illustrations in children's literature that are associated with the language can be particularly helpful for English language learners. When students see the written term, hear it spoken, and view a picture of the term, they are able to make connections to learn the term effectively.

Book	Synopsis
Anno, Masaichiro and Mitsumasa Anno. 1983. *Anno's Mysterious Multiplying Jar*. New York: Philomel Books.	A jar contains an island with two countries. Each country contains three mountains. Each mountain contains four walled kingdoms. The patterns continue until there are 10! of them. Through the book, students learn the meaning and symbols for factorial (!).
Burns, Marilyn. 1994. *The Greedy Triangle*. New York: Scholastic.	A triangle is unhappy with its life and changes into other polygonal shapes. Problems arise when the shape becomes a circle. Eventually, the shape returns to a triangle.
Christaldi, Kathryn. 1996. *Even Steven and Odd Todd*. New York: Scholastic.	Steven only likes even things and Todd only likes odd things. The book can help students understand these concepts.
Crosbie, Michael J. 1993. *Architecture Shapes*. New York: John Wiley & Sons.	Geometry shapes and vocabulary are introduced through architectural pictures.
Ellis, Julie. 2004. *What's Your Angle, Pythagoras?* Watertown, MA: Charlesbridge.	A brief biography of Pythagoras includes an introduction of the Pythagorean theorem, with pictures to illustrate the concept underlying the theorem.
Hoban, Tana. 1974. *circles, triangles, and squares*. New York: Simon & Schuster.	Photographs of real-world objects introduce these shapes.
McCallum, Ann. 2006. *Beanstalk: The Measure of a Giant*. Watertown, MA: Charlesbridge.	In this take on the familiar Jack and the Beanstalk tale, the focus is on aspects of ratio as Jack compares himself to a giant boy named Ray.
McMillan, Bruce. 1991. *Eating Fractions*. New York: Scholastic.	Pictures show food cut into various fractional parts.

FIG. 9.10
Sample children's literature books to introduce mathematics language or symbols

Book	Synopsis
Neuschwander, Cindy. n.d. *Sir Cumference and the Dragon of Pi: A Math Adventure*. Watertown, MA: Charlesbridge.	Sir Cumference must determine the meaning of pi to save his father.
Neuschwander, Cindy. 1997. *Sir Cumference and the First Round Table: A Math Adventure*. Watertown, MA: Charlesbridge.	Sir Cumference helps develop a round table for the knights and invents the terms *radius* and *diameter* to describe the characteristics of a circle.
Neuschwander, Cindy. 2004. *Sir Cumference and the Great Knight of Angleland*. Watertown, MA: Charlesbridge.	Sir Cumference learns about angles on his adventure.
Neuschwander, Cindy. 2006. *Sir Cumference and the Isle of Immeter*. Watertown, MA: Charlesbridge.	Area and perimeter are introduced as the characters try to solve a riddle.
Pallotta, Jerry. 2002. *Apple Fractions*. New York: Scholastic.	Different types of apples cut in various ways introduce fractions.
Schwartz, David M. 1998. *G Is for Googol: A Math Alphabet Book*. Berkeley, CA: Tricycle Press.	Mathematics terms are related to letters of the alphabet.
Stevens, Janet. 1999. *Cook-a-Doodle-Doo!* San Diego: Harcourt Brace.	A modern spin is given to the tale of the Little Red Hen. The book provides a humorous account of misconceptions in language while the characters make a strawberry shortcake from a recipe.
Sundby, Scott. 2000. *Cut Down to Size at High Noon: A Math Adventure*. Watertown, MA: Charlesbridge Publishing.	Concepts related to ratios and scale drawings are introduced through a story about hairstyles in the Wild West.

FIG. 9.10
Continued

Summary

As teachers, we need to heighten our sensitivity and that of our students to the language of mathematics. We need to be aware of when and how new terms are introduced. We need to be alert to terms that appear in everyday English and help students make distinctions between the uses of the term in English and mathematics, and where possible, see the source of the commonalities and differences. We need to make our mathematics classes language intensive to develop mathematics language learners.

Expand Your Understanding

1. Think of several mathematics terms at the middle grades level important in each of the following content strands. For a few of the words, identify issues that might make the terms a challenge for middle grades students to learn.
   - Number and operations
   - Geometry
   - Measurement
   - Patterns and algebra
   - Data analysis and probability
2. a. Identify some confusing pairs of words from each of the content strands in question 1.

   b. For a pair of related words, describe how you might dramatize for students the differences between them.

   c. Create a poster or computer graphic to help clarify the distinctions between a commonly confused pair of words.
3. Research the etymological roots of a few words in each content strand that are of interest to you.
4. Review your mathematics textbook (or a textbook in another discipline). Determine whether terms tend to be introduced before or after concept development.
5. For a word that has both a mathematical meaning and a less technical meaning in English, create a Venn diagram to clarify the meanings that the words have in common. Identify the distinctions between the words.
6. Compare and contrast *Distinguished Pairs* with the *These Are/These Are Not* charts from Chapter 8. When might you choose to use each one?
7. Identify some other children's or adolescent literature that has a mathematics aspect and can be used to help students understand mathematics language or concepts.

Connect to Practice

8. Reflect on the strategies introduced in this chapter. Which ones do you feel confident using? Explain.

9. What strategies do you already use to support students' mathematics language acquisition and fluency? In what ways are these strategies helpful?

10. Consider each of the following vocabulary challenges experienced by a middle grades student. What might be the source of the difficulty for the student? How might you help the student address the challenge?

a. A student thought a *mixed number* was a reciprocal because the two parts were mixed up.

b. A student thought an *expression* was how your face looked.

c. Some students thought an *inequality* was a false equality.

d. Some students thought the *difference* between 29 and 92 was that one is larger than the other.

11. Before being introduced to formal terms, one class invented terms for the median of the lower half of the data (first quartile or 25th percentile): small median, little median, or junior median.

a. How might these invented terms support students' sense-making efforts?

b. What might be comparable invented terms for the third quartile or 75th percentile?

12. Many students need to have visual images of concepts to help them make sense of the concept. Suppose your students struggled to find the total of 16 items whose *average* price was $1.14. How might you help students visualize an average?

13. Preview the curriculum materials that your students will study in the next few weeks.

a. What terminology will students confront?

b. What practices might you use to support their language development?

14. Listen closely to students over the next week and identify instances in written or oral work where students might benefit from support in language development.

a. Discuss these instances with colleagues.

b. What strategies might you use to help these students address their difficulties with language?

15. Work with a colleague to plan some language development strategies for a unit in the future. Implement your plan and evaluate its effectiveness.

Being mathematically literate requires an ability to integrate several literacies in a logical way. In order to be mathematically literate, one must have the essential basic and intermediate literacies (Shanahan & Shanahan, 2008, pg.Â There are many aspects to mathematical literacy, in this paper we looked at what are the two most important. It is the ability to translate a real world problem into a mathematically solvable problem and the ability to use current theory to create new theory that makes one mathematically literate.