Commonalities in the Liquidity of a Limit Order Book ⋆

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Abstract

This paper investigates the commonality of liquidity for an electronic limit order market. We use order book data from the trading facility for German equities. We construct measures of order book liquidity by aggregating the liquidity supply in the book and by estimating the shape of the price impact function. The data provide evidence of substantial common movements in liquidity. Commonalities are much stronger for the complete liquidity supply as opposed to liquidity at the best price alone. Furthermore, commonalities vary with the time of the day and with market momentum, yet we find no significant industry factor.

Key words: Order-Driven Markets, Liquidity Commonalities
JEL: G10, G14

1 Introduction

In asset pricing theory, the expected return of a stock is related to its sensitivity to the fluctuation of certain state variables. Previous studies such as Amihud (2002), Pástor and Stambaugh (2003) or Gibson and Mougeot (2004) provide empirical evidence that market-wide liquidity is such a priced state variable. While this implies that liquidity risk has a systematic component, the evidence of common liquidity fluctuation is very weak. Most prominently, Chordia, Roll and Subrahmanyam (2000) observe that less than 2% of a stock's liquidity variation is market-wide. This is a surprising discrepancy: on the one hand, liquidity risk obviously gets priced, yet on the other hand it seems to be diversifiable across assets.

One possible explanation for these puzzling results is that previous studies of commonality rely on liquidity around the best bid and ask alone. There are two strong arguments against this approach in today's anonymous limit order book markets. Firstly, on a more conceptual level, liquidity at the best limit prices only represents a fraction of the limit orders in the order book. Traders consume and supply far more liquidity than there is available at the best price limits. Secondly, on a more technical level, best limit prices are exposed to idiosyncratic shocks very heavily and attract a lot of noise. For example, undercutting strategies lead to permanent variation of the spread and its corresponding depth because small new limit orders with price priority are submitted all the time. In this paper, we extend the concept of liquidity to the entire order book and investigate to what extent liquidity fluctuations really are market-wide.

While the pricing of liquidity risk is an implication of market-wide fluctuations, the presence of commonality also has direct relevance for investors as well as markets as a whole:

(1) At the level of the individual market participant, systematic liquidity risk implies that investors who wish to satisfy a large cash demand cannot readily switch from one portfolio position to another to bypass illiquid assets. The larger the investor's portfolio positions to be purchased or sold, the further the transactions walk up the book and the more relevant the exposure to systematic liquidity risk becomes.
(2) At the level of the entire market, an exchange depends crucially on the provision of new liquidity after the order book has been cleared. If liquidity is provided and withdrawn in common patterns across assets, the whole market will exhibit systematic fluctuations. If this is the case, exchanges and regulators will have to keep an eye on the cross-sectional impact of liquidity shocks in order to guarantee stability.

The literature on common liquidity risk is not that large yet. Chordia, Roll and Subrahmanyam (2000) introduced the idea of market-wide liquidity in an empirical paper that uses daily US quote data. They provide evidence that their index of market liquidity has a significant impact on individual stock liquidity. Brockman and Chung (2002) apply this approach to intraday data from Hong Kong's purely order-driven stock market and obtain very similar results. Halka and Huberman (2001) document correlation in the liquidity of different stock portfolios. Hasbrouck and Seppi (2001) on the other hand find that little commonality remains once time-of-day effects in the liquidity levels have been taken into account. In all, the literature does provide some first evidence that commonality in liquidity exists, yet its low level does not make it very persuasive. [1]

The most obvious shortcoming of the empirical literature is that it only considers best bid and best ask quotes. Our study will show that this is only a tiny fraction of the liquidity that is supplied in the limit order book. We use data from the Xetra trading system of the Frankfurt Stock Exchange (FSE) which enables a flawless reconstruction of the order book at any time. We will proxy order book liquidity by aggregate depth in the book and by the shape of the price impact function. Our study departs from the shortcomings of the previous literature to give a comprehensive characterization of the commonality of liquidity. In particular, we address the following questions in this paper:

- Does liquidity exhibit market-wide fluctuations?
- Does commonality depend on order book volume?
- What determines market-wide liquidity risk?
- Does the evidence depend on the methodology used?

Our study yields four central results: firstly, we find evidence of significant common variability in liquidity throughout the order book. Secondly, commonality increases with order book depth: at best prices, common variation in aggregate depth only accounts for roughly 2% of overall variability, while it increases dramatically for larger order book depth (up to levels as high as 20%). Thirdly, commonality is not driven by industry factors, whereas market momentum and the time of the day do have a considerable impact. Finally, these results are stable and qualitatively identical for a regression model approaches as well as principal component analyses. Benchmarking the results against return covariation, commonality in order book liquidity reaches about 20%, while commonality in returns is about 32% for the same sample. These

[1] On the theoretical side, the interaction of liquidity and returns has been modeled by Amihud and Mendelson (1986) or, more recently, by Acharya and Pedersen (2003). These approaches take systematic liquidity as granted. The newer empirical findings have now led to first attempts at modeling the emergence of commonalities (see Fernando (2003) or Watanabe (2003). However, these models are still far from being applicable to the structure modern limit order markets.

high figures reinforce that commonality in order book liquidity is too strong to be neglected any further.

The remainder of the paper is organized as follows: In Section 2 we present the market architecture of our data set, the central properties of the sample stocks and we eliminate trends from the data. Section 3 investigates the commonality of best price liquidity as a reference scenario. In Section 4 we construct measures of order book liquidity and estimate the extent of market-wide fluctuation. We then perform a principal components analysis to compare the impact of the methodology on the results. Section 5 investigates the influence of stock industry, time of day and market momentum. Section 6 concludes.

2 Market Structure, Data and Liquidity Measures

Our study uses data from the electronic limit order market at the Frankfurt Stock Exchange (FSE). The electronic system used in Frankfurt is called Xetra and allows trading to anyone who is connected to the Xetra computer system. The same trading platform is also used at the stock exchanges in Vienna and Dublin. Although trading through Xetra at the Frankfurt Stock Exchange does face some competition from regional exchanges or floor trading facilities, Xetra accounts for roughly 98% of all trading activity; floor trading and regional exchanges pale to insignificance.

FSE operates as an open limit order book. Trading is based on a continuous double auction mechanism with automatic matching of orders. Trading hours at FSE extend from 9.00 to 17.30 CET. In this time, the order book fully displays all orders that are submitted to the market. This is a considerable difference compared to other systems like NYSE where only specialists can access the complete book or Paris where only the best five orders are displayed. For some small-cap stocks listed in Frankfurt there are specialists who provide a minimum level of liquidity. Yet the blue-chip segment of FSE operates without any market makers and purely relies on the anonymous submission of limit orders. While the book is open, it gives all limits, the accumulated order volumes at each limit and the number of orders in the book at each limit. [2]

The basic order types allowed during continuous trading are conventional market orders, limit orders as well as market-to-limit orders. [3] Each new incoming

[2] The continuous trading phase begins after an opening auction at the beginning of the day and it is closed by a closing auction at the end of the day. During auctions, the order book is closed, yet during continuous trading, the whole limit order book is visible to market participants.

[3] A market-to-limit order is treated as a market order and executed against the best price in the order book. However, the remaining part that cannot be executed

Table 1
Summary Statistics of the Data Set

	Mean	Median	Max	Min
Market capitalization	18.20	10.39	61.29	2.95
Stock return (logarithmic)	-1.06	-0.54	18.80	-20.89
Average daily trading volume	114.79	75.22	348.60	14.13
Absolute bid-ask spreads	0.04	0.03	0.09	0.01
Relative bid-ask spreads	0.09	0.10	0.15	0.05
Depth (at the best limit price)	180,487	148,461	979,214	65,773

Table 1 summarizes the main characteristics of the stocks in our data set. Market capitalization is given in billions of Euros and log returns in %. Trading volume is the average daily trading volume in millions of Euros. Absolute spreads and depth at the best price are in Euros and relative spreads in %.

order is immediately checked for execution against orders on the other side of the order book. Matching and price determination take place on the basis of price priority and time priority. The time criterion applies in the event of orders sharing the same limit, i.e. orders entered earlier take priority. Furthermore, market orders have priority over limit orders. Incoming market orders are executed at the highest bid limit or lowest ask limit and, in the case of large volumes, walk up the book. It is also worth noting that FSE has no tick size restriction. The tick size is 1 Eurocent (0.01 Euros) which corresponds to the currency's smallest possible value. Its transparency and its sole reliance on anonymous limit order submissions make this trading design well suited for the study of the microstructure of markets and agents' behavior. [4]

The actual data set we use consists of the thirty equities which make up the blue-chip index DAX 30. It ranges from 2 January 2004 and to 31 March 2004. The raw data is the computerized trading protocol in which FSE keeps track of all entries, cancellations, revisions, executions and expirations of orders. Additionally, the stock exchange recorded the initial state of the order book at the beginning of our data range. From there, we track each order entry and perform an ongoing reconstruction of the order book by implementing the Xetra market model. We update the order book for each order book event and end up with a huge sequence of order books from which we take snapshots at 30-minute intervals. We exclude order books which have been preceded by an auction, since the supply mechanism for liquidity is different in auctions. Finally, we end up with 17 order books every day.

Table 1 gives some summary statistics. The stocks' market capitalizations range from 2.95 billion Euros to 61.29 billion. Average daily trading volume

at the best price is converted to a limit order at the transaction price of the market order part.

[4] For more details on the Xetra market model at FSE see Deutsche Boerse Group's manual of their stock trading models.

varies between 14.1 million Euros and 348.6 million Euros. Absolute spreads (best ask price minus best bid price) lie between 0.01 and 0.09 Euros. Relative spreads (absolute spreads divided by the midquote) vary between 0.05% and 0.15%. The average depth, which is the number of shares on the ask and bid side multiplied by the respective best ask and bid, lies between 65,773 and 979,214 Euros. These figures show that liquidity for blue chip stocks exhibits considerable cross-sectional variation. Yet on average, the level of liquidity seems to be fairly high. About 97.3 % of all submissions are limit orders and only 2.1 % are market orders. Of the submitted limit orders, 23 % get executed and 77 % cancelled. These figures show that the liquidity consumed by transactions is only a fraction of the liquidity provided to the book. Furthermore, the data allows to compute how far market orders walk up the book: about 80% of all transactions take place within the best prices, while the remaining 20% consume more liquidity. Such transactions can actually be very large and walk up the book more than 100 ticks.

The previous literature such as Wood, McInish and Ord (1985), Jain and Joh (1988), Foster and Viswanathan (1990) or McInish and Wood (1992) documents that liquidity shows strong seasonal patterns, in particular on an intraday basis. We therefore plot time-specific averages of the spread and the corresponding depth against the time of day. Fig. 1 shows that, consistent with previous empirical evidence, we observe the well-known U-shaped pattern for spreads and an increase of depth. Since the level of spreads is nearly twice the size at the beginning and end of the trading day, we fear that differencing alone will not remove time-of-day patterns if a higher level of liquidity is equivalent to a higher standard deviation. Secondly, if liquidity does not exhibit unit-root behavior, we would be inducing serial correlation in residuals.

Like Hasbrouck and Seppi (2001) we standardize liquidity by time-specific means and standard deviations. This also makes sense from an economic point of view because only the unexpected component of liquidity variation constitutes risk. Commonality due to time-of-day effects is not risky. By standardizing we indirectly eliminate the proportion of commonality induced by the time of day alone. Let p denote the time of day k for all trading days n. For all points in time and for every stock, we calculate the time-specific means, $\mu_{j,p} = \Sigma_{k=1}^{n} LP_{j,p,k}/n$, and standard deviations, $\sigma_{j,p} = \sqrt{\Sigma_{k=1}^{n}(LP_{j,p,k} - \mu_{j,p})/n}$. Thus, we obtain 17 means and standard deviations for all stocks, one such pair for every point in time. Let an asterisk denote a variable of stock j adjusted for trends. We then demean and standardize all observations according to their point in time and obtain new liquidity proxies:

$$LP^{*}_{j,p,k} = \frac{LP_{j,p,k} - \mu_{j,p}}{\sigma_{j,p}} \tag{1}$$

This procedure is applied both to aggregate order book depth measures as

Fig. 1. Intraday Pattern of Average spreads and Average Depth

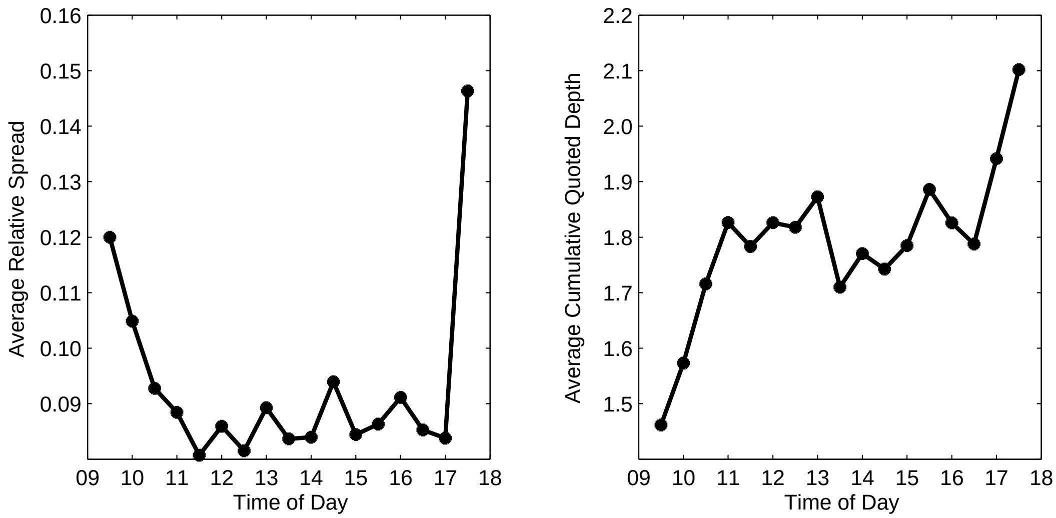

Figure 1 shows typical time-of-day effects for spreads and cumulative depth. The x-axes give the time of day and the y-axes the average relative spread (in %) or the average depth (in millions of Euros). Spreads exhibit the typical decreasing pattern, dollar depth displays the typical increasing pattern.

well as the parameter estimates of the price impact function. These measures of unexpected liquidity constitute the input for the analysis of commonality.

3 Commonality of Best Price Liquidity

Previous studies have implicitly assumed that best limit prices alone are sufficient to capture the liquidity of an asset. To relate our results to the literature, we analyze the commonality of liquidity for the depth of the order book at the best bid and ask limit prices.

The standard econometric approach is the simple market model in Chordia, Roll and Subrahmanyam (2000) which is estimated by time series regressions. The market model measures the sensitivity of stock j's liquidity, L_j, to market liquidity, L_M. Market liquidity is computed as the average liquidity across all stocks, $\Sigma_{j=1}^{30} L_{j,t}/30$, where t is a time subscript.[5] The estimation includes lead and lag market liquidity ($L_{M,t+1}$ and $L_{M,t-1}$), contemporaneous, lead and lag market returns ($r_{M,t}$, $r_{M,t+1}$ and $r_{M,t-1}$) as well as individual stock return volitility $vol_{j,t}$ (proxied by the squared return) as additional regressors. With ε as an error term, we obtain the following specification:

[5] For each stock j's regression, stock j is dropped in the calculation of market liquidity. Previous studies document that the results are unchanged for a value-weighted market liquidity measure.

Table 2
Market Model for Depth at the Best Limits

		Average Parameter	Average t-value	% Significant Parameters
Intercept	α	0.12	-3.61	77
Market Liquidity	β^1	0.27	2.51	70
	β^2	0.14	0.67	17
	β^3	0.13	1.03	30
Market Return	δ^1	0.01	-0.05	10
	δ^2	0.08	0.40	17
	δ^3	-0.12	-0.29	33
Return volatility	η	0.00	0.09	17
Adjusted R^2			0.02	

Table 2 reports the parameter estimates of the liquidity market model (Equation 2) for spreads and depths. It gives the mean parameter estimates across all 30 stocks, the average t-values and the percentage of significant coefficients at the individual stock level (in %) for both spreads and depths. The last row reports the average adjusted coefficients of determination of the regressions.

$$L_{j,t} = \alpha + \beta_j^1 L_{M,t} + \beta_j^2 L_{M,t+1} + \beta_j^3 L_{M,t-1} + \delta_j^1 r_{M,t} + \delta_j^2 r_{M,t+1} + \delta_j^3 r_{M,t-1} + \eta_j vol_{j,t} + \varepsilon_{j,t} \qquad (2)$$

Tab. 2 summarizes the estimation results of equation (2) for liquidity at the best limit prices.[6] For depth at the best bid and ask, β^1 has an average of 0.27 across stocks and an average t-values of 2.51. It is positive in 100% of the time and significant in 70%. The average t-values for the coefficients of the additional regressors are insignificant and individual coefficients are significant only very seldom. The average adjusted R^2 measure of the regressions is 2%. It can be interpreted as a measure of commonality, since it explains the percentage of individual liquidity variation that is explained by market liquidity.[7] Thus, commonality in depth is significant, yet its contribution to overall depth variation is small at a level of 2%.

The market model results that we obtain are very much along the same lines as in the literature. As in Chordia, Roll and Subrahmanyam (2000) and Brock-

[6] We average the estimated parameters and their corresponding t-values across all stocks as in Chordia, Roll and Subrahmanyam (2000) and Brockman and Chung (2002). Since the variation is not that strong and the sign is always the same, this should be a quite accurate summary statistic. The cross-sectional t-statistic implicitly assumes that the estimation errors in β_J are independent across stocks.

[7] Strictly speaking, it captures the effect of all explanatory variables, not only market liquidity. However, leaving out the insignificant regressors does not lower the adjusted R^2 values very much.

man and Chung (2002), only the market liquidity parameter is significant, while the additional regressors are not. They report adjusted R^2 values of 1% and 2.1%, while we also obtain a value of 2% for the depth at the best limit price. Unlike the literature, we find no systematic size factor at work in our results. The market depth coefficient increases with firm size, however the adjusted R^2 value is highest for the portfolio with the lowest coefficient. We should not be too troubled by the absence of a size factor since it has no real theoretical justification in the first place.

4 Commonality of Order Book Liquidity

In this section we extend commonality to the order book. First, we construct measures of order book liquidity and secondly compute the extent of commonality they show. Finally, we employ principal components analysis (PCA) to check that the results are stable with respect to the methodology.

4.1 Measures of Order Book Liquidity

In traditional specialist markets, traders typically only saw the buy and sell price that a market maker quotes for a stock. Large trades often involved a different trading mechanism like the so-called "upstairs market making". In these dealer markets, the spread seems to be an adequate description of stock liquidity. In limit order book markets however, all orders are executed against the limit orders in the order book. Fig. 2 shows that depth at the best price alone is only a small fraction of the order book's total depth. To get a more precise view of order book liquidity, we construct a measure of aggregate order book depth.

Let x denote order book volume (number of shares multiplied by midprice) and let $MP_i(x)$ denote the marginal price of the last unit of this volume. Since prices are non-stationary and differ in levels across stocks, we transform prices into price impacts. We define price impacts RI relative to the prevailing midquote and dependent on volume: [8]

$$RI_i^A(x) := \frac{MP_i(x) - MQ_i}{MQ_i} \tag{3}$$

Thus we can convert each price limit in the order book into a price impact

[8] Relative price impacts on the bid side require the switching of marginal prices and midquotes in the numerator.

Fig. 2. Typical Order Book

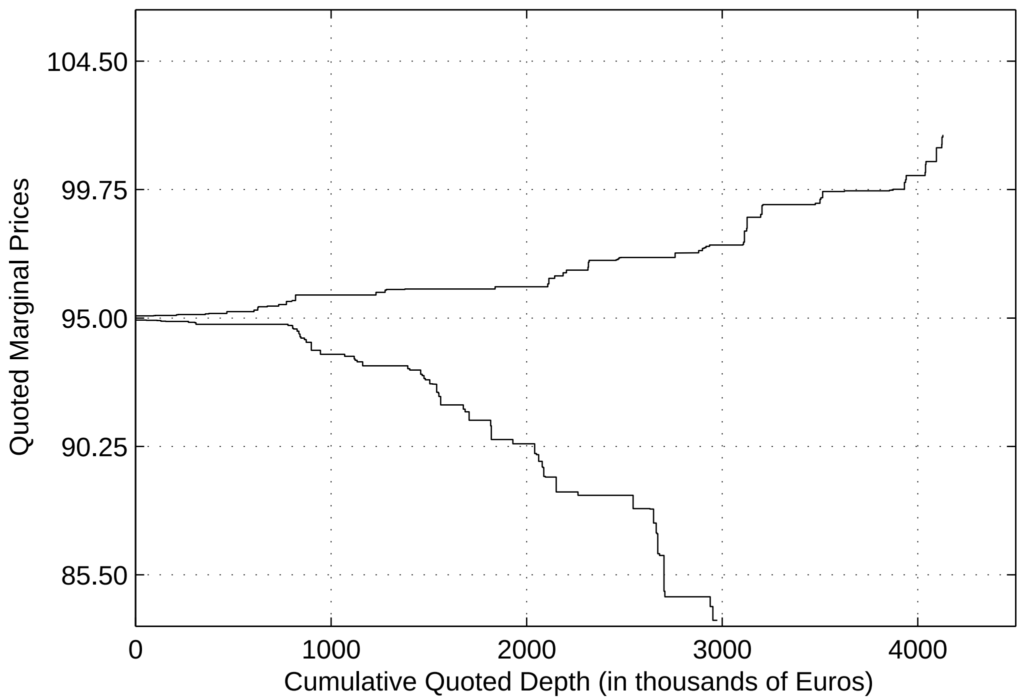

Figure 2 shows an order book for Adidas. The horizontal axis gives cumulative depth and the vertical axis depicts the corresponding marginal prices. The empirical price impact function is a step function.

which is relative to the midquote. We aggregate all depth from the best price limit to given steps of the order book function to determine the corresponding aggregtate order book depth.

Actual price impacts that market orders incur will very much depend on the order size and the liquidity of the stock. In our sample of liquid stocks, maximum price impacts of market orders are slightly below 2%. For smaller and less liquid US stocks, Keim/Madhavan (1997) document price impacts up to 8%. To characterize order book liquidity for different trade sizes and different order book regions, we sample aggregate order book depth at 0.5%, 1%, 1.5%, 2% and 5% price impacts. This measure is computed for the ask side and the bid side of the book separately. [9]

A second measure of order book liquidity addresses the shape of the price impact function. The empirical literature on price impacts comes to very mixed conclusions as to the shape of the price impact function. Biais, Hillion

[9] This measure of liquidity is very similar to the cost of round trip in Benston, Irvine and Kandel (2000), the XLM measure in Gomber, Schweickert and Theissen (2004) or hypothetical price impacts as in Kumar (2003). We use marginal unit prices instead of average prices, because utility-optimizing investors equate marginal utility and marginal costs. Marginal prices thus seem to be the more natural candidate. Furthermore, unlike cost-of-round-trip measures, our measure captures asymmetries of the bid side and the ask side.

and Spatt (1995) document linearity, while Coppejans, Domowitz, Madhavan (2003), Benston, Irvine and Kandel (2000) and Cao, Hansch and Wang (2003) find evidence of non-linearities. According to Griese and Kempf (2004), the price impact function is concave in 50% of the time and convex otherwise. In the absence of clear guidance from the literature, we restrict our efforts to simple and easily tractable specifications. Let m denote the price-volume combinations of the individual regressions, let t denote the individual points in time at which the regression equations are estimated and let i denote the stocks. If ε is the error term, we obtain the following equation to estimate a linear model price impact function:

$$RI_{itm} = \alpha_{it} + \beta_{it} \cdot D_{itm} + \varepsilon_{itm} \tag{4}$$

In the estimation procedure, we cut off the price impact function for price impacts higher than 2%. Limit orders beyond this cut-off value tend to bias the slope estimates upwardly, yet they correspond to such high volumes that they never get executed. We estimate the model for each stock at every point in time to obtain a time series of parameter estimates. Since the price impact function is upward-sloping by construction, it is not surprising that the fits turn out to be very good and that the parameters estimates are highly significant. The average in-sample adjusted R^2 is 0.92. The average t-values of the slope parameters are significant for all 30 stocks with a cross-sectional average of 24.01.[10] The estimates and significance on the ask side and bid side are qualitatively identical.

4.2 Aggregate Order Book Depth

In this section we investigate market-wide liquidity movements of the entire limit order book. We use the same methodology as before, yet this time we substitute $L_{j,t}$ and $L_{M,t}$ in Equation 2 by our measures of order book liquidity. Our special interest lies in the link between commonality and aggregate order book depth as well as the extent of commonality compared to the reference model in the previous section.

Table 3 gives the results for order book depth at 0.5% price impact. The beta coefficient for contemporaneous market liquidity is the only coefficient with an average t-value that is significant both on the ask side and the bid side. On

[10] If we wanted to compare the performance of different models, we should be looking at their predictive power and thus out-of-sample R^2 values. Since this is not the focus of our paper, we refer the reader to the relevant literature. Diebold/Li (2002) use such an approach for the estimation of yield curve. Griese/Kempf (2004) and Bowsher (2004) apply the approach to price impact functions.

Table 3
Market Model for Aggregate Depth up to 0.5% Price Impact

		Ask Side			Bid Side		
		Mean	t-value	%	Mean	t-value	%
Intercept	α	-0.01	-0.30	7	0.00	-0.03	0
Market Liquidity	β^1	0.53	3.19	97	0.46	2.74	73
	β^2	0.12	0.87	27	0.12	0.78	33
	β^3	0.14	0.98	37	0.12	0.79	27
Market Return	δ^1	0.18	1.18	30	-0.15	-0.97	33
	δ^2	0.05	0.36	10	-0.04	-0.27	10
	δ^3	-0.08	-0.49	20	0.02	0.16	10
Return volatility	η	0.04	0.92	33	0.02	0.56	37
Adjusted R^2			0.09			0.06	

Table 3 reports the parameter estimates of the liquidity market model (Equation 2) for aggregate order book depth up to a price impact of 0.5%. It gives the mean parameter estimates across all 30 stocks, the average t-values and the percentage of significant coefficients at the individual stock level (in %) for the ask side and the bid side of the order book. The last row reports the average adjusted coefficients of determination of the regressions.

the ask side, β^1 is positive in 100% of the regressions and significant in 97% of the cases. On the bid side side, it is also positive in 100% of the estimated equations and significant in 73% of the time. Like before, the average t-values of all other regressors indicate that they are not significant. Compared to the results of liquidity at the best limit price alone, however, the adjusted R^2 values are much higher. They climb to an average of 9% on the ask side and 6% on the bid side as opposed to 2% before. In other words, commonality already increases if we consider liquidity provided to the book at a few ticks from the best limit price.

Table 4 shows how commonality changes if we increase order book depth. On the ask side, β^1 starts out at 0.53 and increases continually to 0.72 for order book depth at a 2% price impact. At a price impact of 5%, it has a value of 0.85. The corresponding average t-values are very clearly above the critical value of 1.65. Turning to the level of commonality, we observe that the adjusted R^2 value on the ask side is 9% for a price impact of 0.5%. It increases continually up to 19%. This percentage is astoundingly high if we remember that commonality for depth at the best prices was only 2%. The pattern holds for the bid side of the order book as well where commonality increases to a value of 11%. [11]

[11] Like in the previous section, we regroup the stocks according to size portfolios and check for systematic size effects. Again, we find no evidence of any clear size patterns and thus do not report the results in detail. Firm size does not seem to bias the commonality of liquidity in any significant manner.

Table 4
Market Model for Increasing Order Book Depth

	Ask Side			Bid Side		
Price impact	β_1	t-value	R^2	β_1	t-value	R^2
0.5%	0.53	3.19	0.09	0.46	2.74	0.06
1.0%	0.53	2.50	0.12	0.49	2.42	0.07
1.5%	0.67	3.33	0.14	0.62	3.22	0.08
2.0%	0.72	3.72	0.16	0.69	3.70	0.08
5.0%	0.85	4.71	0.19	0.80	4.56	0.11

Table 4 reports parameter estimates of the liquidity market model (Equation 2) for increasing aggregate order book depth. The first column lists the price impact up to which depth is aggregated. The further columns give average parameter estimates, corresponding t-values and average coefficients of determination for the ask side and the bid side of the order book.

Next we move to the shape of the price impact function. Table 5 presents the results for the slope parameter estimates of the linear model of the price impact function. The market liquidity coefficient is 0.77 on the ask side and 0.72 on the bid side and are both highly significant. All other coefficients are close to zero in a range between -0.07 and 0.07 with small and insignificant t-values. On the level of the individual stocks, 93% of the individual market liquidity coefficients are significant (for the ask and bid side slope parameters). Commonality on the ask side is 16% and on the bid side 10%. Evidently, the results for slope parameters of the price impact function are very similar to those for aggregate depth.

The results are illustrated in Fig. 3: if liquidity is measured by best limit prices alone, the proportion of market-wide changes is very small at about 2%. However, in a limit order book market we should measure liquidity as the limit orders in the book. If we thus aggregate the available depth, we see that commonality rises sharply. The most plausible explanation for the strong rise is that best limit prices are clouded by a lot of noise from stock-specific trading strategies. If the amount of depth that is aggregated rises, stock-specific noise becomes relatively small and a clearer pattern of commonality emerges. To put these results into perspective, we rerun our calculations for the returns of our sample stocks and obtain a market-wide component of return fluctuation of 32%. In comparison, market-wide liquidity movements on the ask-side are about two thirds the level of market-wide return covariation and one third on the bid side. In the light of such high liquidity comovement, it is not that puzzling anymore that there is evidence that liquidity risk gets priced.

Table 5
Market Model for the Shape of the Price Impact Function

		Ask Side			Bid Side		
		Mean	t-value	%	Mean	t-value	%
Intercept	α	0.00	0.06	0	0.00	0.09	0
Market Liquidity	β^1	0.77	5.23	93	0.72	4.59	93
	β^2	0.04	0.31	13	0.02	0.18	33
	β^3	0.06	0.46	30	0.03	0.28	47
Market Return	δ^1	-0.07	-0.50	20	0.03	0.24	13
	δ^2	-0.04	-0.28	23	0.07	0.47	13
	δ^3	0.02	0.12	20	0.00	0.04	13
Return volatility	η	-0.01	-0.31	13	-0.02	-0.38	13
Adjusted R^2			0.16			0.10	

Table 5 reports the parameter estimates of the liquidity market model (Equation 2) for the slope parameter of the price impact function. It gives the mean parameter estimates across all 30 stocks, the average t-values and the percentage of significant coefficients at the individual stock level (in %) for both the bid side and the ask side of the order book. The last row reports the average adjusted coefficients of determination of the regressions.

4.3 Principal Components of Liquidity

In the following section we approach commonality from a different angle. While Chordia, Roll and Subrahmanyam (2000) directly assume that a market average explains individual stock liquidity, other studies such as Hasbrouck and Seppi (2001) or Hansch (2001) use principal component analysis (PCA), a more statistical approach. We compare the results of these two methodologies for our data and conclude to what extent they influence the findings.

The main input of PCA is the covariance matrix of a group of random variables. In our case, these random variables will be the sample stocks' liquidity measures. If there are differences in the level of these variables, the correlation matrix is more suited. PCA then extracts linear combinations of all individual variables which explain the highest proportion of variability in the data. These linear combinations, which are called principal components, are contained in the eigenvectors of the correlation matrix and their explanatory power in the corresponding eigenvalues. Since principal components are orthogonal by construction, the first eigenvalue will typically be high and further eigenvalues low. Once we have determined the principal components, we regress them onto our liquidity time series for each stock and bootstrap a test statistic from the residuals. Suppressing firm subscript j, we denote the realization of the first principal component in t by x_t. Let ξ and ψ be parameters and ε an error term. We perform the following regression analysis,

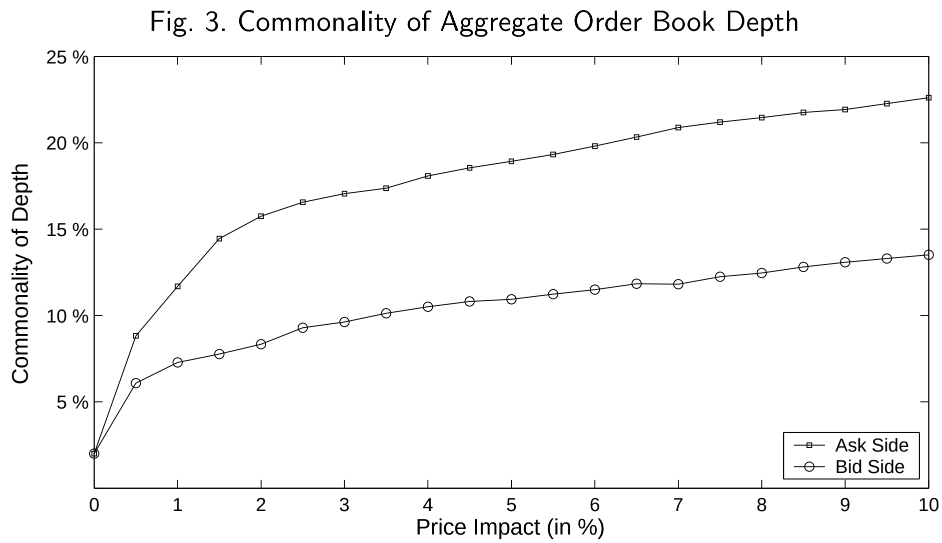

Figure 3 shows how the extent of commonality increases as more liquidity in the order book is considered. The x-axis gives the price impact up to which depth is aggregated and the y-axis gives the amount of market-wide movement.

$$L_t = \xi + \psi \; x_t + \varepsilon_t, \tag{5}$$

and keep the time series of residuals ε for each stock. To generate a test statistic, we simply bootstrap new time series from the regression residuals of all stocks, compute the correlation matrix and perform PCA. We repeat this procedure 10,000 times until we obtain a smooth empirical distribution of the first eigenvalue and sample the 95%-quantile as our critical value. Bootstrapping has the advantage that it preserves the original distribution of the regression residuals and imposes no distributional assumptions such as normality. [12]

Table 6 summarizes the PCA results for aggregate depth. On the ask side, the first principal component of aggregate depth at 0.5% price impact is 4.0. With a critical value of 2.1 it is clearly significant and accounts for 13.3% of overall variation in depth. If we successively increase aggregate depth up to 2%, the amount of common variation rises continually from 13.3% to 20.1%. At 5% price impact, common variation is 23.3%. All first principal components remain strongly significant. [13] On the bid side of the book, the pattern is identical. All first principal components for aggregate depth are significant. In

[12] Critical values for the second principal component are obtained in exactly the same way, yet we have to regress onto the first two principal components first before bootstrapping from the residuals.

[13] Although all additional regressors turned out to be insignificant in the previous market average approach, we eliminate their impact on our liquidity measures to doublecheck the significance of our results. In a first step, we regress the liquidity measures onto the same explanatory variables, then compute the correlation matrix of all 30 stocks from their residuals and finally repeat the PCA procedure. This robustness check leads to slightly lower levels of commonality, yet the first principal components remain significant. All second principal components are insignificant.

Table 6
PCA Results for Aggregate Order Book Depth

	Price Impacts of Depth				
PCA output	0.5%	1.0%	1.5%	2.0%	5.0%
Ask Side					
First eigenvalue	4.00	4.85	5.69	6.04	6.99
Critical value	2.13	2.53	2.84	2.92	3.69
Proportion of Variation	13.32	16.17	18.97	20.13	23.30
Bid Side					
First eigenvalue	3.24	3.57	3.66	3.80	4.51
Critical value	2.01	2.19	2.48	2.54	3.52
Proportion of Variation	10.80	11.92	12.20	12.66	15.03

Table 6 gives the results of PCA for order book depth aggregated to various different price impact levels. In the first section, the table lists the first eigenvalue, its critical values at the 95% confidence level and the proportion of total variability explained by the first principal component (in %) for the ask side of the order book. The second section gives the same information for the bid side.

level, commonality starts out at 10.8% and rises to 12.7% if we consider depth up to 2% price impacts. For 5% impacts, commonality is about 15%. If we compare these results to the regression approach in the previous section, we see that they are very similar indeed. Firstly, commonality increases with order book depth. Secondly, commonality is stronger on the ask side of the book. In level, the PCA results are about 4% above the market index approach for ask side depth and 2% for bid side depth. Somewhat higher PCA results are not surprising, since PCA is not restricted to a predetermined market average liquidity measure. Without reporting the results for parameter estimates in too much detail (24% commonality on the ask side and 17.9% on the bid side), we again observe the same similarity between the market model approach and PCA.

5 Industry, Time and Momentum

In this section we investigate whether market-wide commonalities in liquidity vary with any further factors. In particular, we analyze the impact of the industry as well as the time of the day and the momentum of the market. To limit the number of tables, we show the empirical findings for different sizes of order book depth.

Table 7
Impact of Industry on Commonality

	Ask Side				Bid Side			
	Market		Industry		Market		Industry	
Price impact	β^1	t-value	γ^1	t-value	β^1	t-value	γ^1	t-value
0.5%	0.52	2.95	0.01	0.06	0.43	2.42	0.04	0.54
1.0%	0.47	2.12	0.04	0.43	0.45	2.10	0.04	0.54
1.5%	0.60	2.77	0.06	0.61	0.57	2.73	0.07	0.85
2.0%	0.66	3.15	0.05	0.51	0.64	3.07	0.06	0.65
5.0%	0.80	3.76	0.10	0.82	0.75	3.45	0.09	0.73

Table 7 illustrates the impact of industry on the commonality of liquidity. It reports average coefficients of market liquidity and industry liquidity for the ask side and the bid side of the order book. The results are reported for order book depth aggregated up various different levels of price impact.

5.1 Industry-wide Commonality

In factor models, industry is a popular factor for stock returns. The same can be done for liquidity. The implicit assumption is that stocks from the same industry show more comovement than stocks across different industries. We estimate our market model of liquidity with additional regressors for contemporaneous, lead and and lagged industry liquidity. We use the official FSE industry classifications to construct industry liquidity in the same way as market liquidity. Again, we exclude the dependent variable from the index. Let L_I denote the industry liquidity that corresponds to each stock i and let γ denote its sensitivity to industry liquidity. Sticking to the same notations as in Equation 2, we estimate the following model:

$$\begin{aligned} L_{j,t} = \alpha + \beta_j^1 L_{M,t} + \beta_j^2 L_{M,t+1} + \beta_j^3 L_{M,t-1} + \gamma_j^1 L_{I,t} + \gamma_j^2 L_{I,t+1} \\ + \gamma_j^3 L_{I,t-1} + \delta_j^1 r_{M,t} + \delta_j^2 r_{M,t+1} + \delta_j^3 r_{M,t-1} + \eta_j vol_{j,t} + \varepsilon_{j,t} \end{aligned} \tag{6}$$

Table 7 summarizes the results for industry liquidity. Because the table is already voluminous enough, we only report the coefficients for contemporaneous market and industry liquidity. The coefficients of market liquidity are still highly significant and have the same level as before. The coefficients of industry liquidity, however, are very low between 0.00 and 0.10. On average, they are clearly insignificant. The highest percentage of significant coefficients is 10.5% on the ask and bid side. We can safely say that in our sample industry liquidity does not influence individual stock liquidity. At first glance, these findings seem to be in contrast to the literature. However, although Brockman and Chung (2002) state that firm liquidity depends on market as well as industry liquidity, only 13.5% of their industry coefficients are significant.

Table 8
Impact of the Time of the Day

	Ask						Bid					
	Open		Day		End		Open		Day		End	
Impact	β_1	R^2	β_1	R^2	β_1	R^2	β_1	R^2	β_1	R^2	β_1	R^2
0.5%	0.57	13	0.46	09	0.52	11	0.52	10	0.42	07	0.39	08
1.0%	0.51	15	0.41	12	0.54	14	0.46	13	0.37	08	0.54	09
1.5%	0.57	17	0.46	15	0.75	18	0.55	14	0.46	07	0.73	11
2.0%	0.56	19	0.47	16	0.82	19	0.60	15	0.51	07	0.81	12
5.0%	0.64	22	0.73	18	0.94	24	0.67	16	0.75	10	0.95	15

Table 8 reports the average coefficients of market liquidity and the corresponding coefficients of determination of the regressions (in %) for the opening of the trading day ("open"), the midday trading period ("day") and the end of the trading day ("end"). The results are reported for the bid side and the ask side of the book.

Chordia, Roll and Subrahmanyam (2000) do not report the percentage of significant industry coefficients, yet even their values for market liquidity only range between 14% and 35%. These previous findings of industry liquidity do not seem to be as unambiguous as they were interpreted in the past. Our data gives clear evidence in favor of a significant impact of market liquidity on firm liquidity and clear evidence against an influence of industry liquidity.

5.2 Time of the Day

As we discussed earlier, previous literature has established that liquidity exhibits strong time-of-day effects. Therefore we investigate whether, beyond intraday patterns of individual firm liquidity, commonality exhibits similar trends. Fig. 1 showed that spreads are particularly high until 11 a.m., while depth is low in that interval. The average level then remains stable for the next hours and changes once more during the last two trading hours of the day. At the end of the trading day, both spreads and depth tend to be higher. Therefore, we split our data into three subsamples: a morning sample (order book liquidity until 11 a.m.), a midday sample (from 11.30 a.m. until 3.30 p.m.) and an evening sample (from 4 p.m. to 5.30 p.m.). We reestimate Equation 2 separately for these three subsamples.

Table 8 summarizes the results. Although time-of-day effects have been eliminated from individual liquidity levels, commonality still shows intraday patterns. For example, the adjusted R^2 value for ask side depth at a price impact of 0.5% is 13% in the morning, drops to 9% in the course of the day and goes back up to 11% in the late trading period. This pattern remains stable for the aggregate depth of higher price impacts. At 5%, commonality is at about

Table 9
Impact of Market Momemtum

	Ask				Bid			
	Up		Down		Up		Down	
Price impact	β^1	R^2	β^1	R^2	β^1	R^2	β^1	R^2
0.5%	0.51	0.10	0.63	0.15	0.31	0.06	0.53	0.15
1.0%	0.49	0.14	0.62	0.18	0.40	0.07	0.56	0.20
1.5%	0.61	0.14	0.74	0.21	0.55	0.09	0.72	0.18
2.0%	0.72	0.15	0.78	0.22	0.63	0.10	0.73	0.14
5.0%	0.80	0.21	0.92	0.31	0.76	0.12	0.85	0.21

Table 9 reports the average parameter estimates of the market liquidity parameter and coefficients of determination for rising markets ("up") and falling markets ("down"). It lists the results for the ask side and the bid side of the order book and also differentiates with regard to the price impact up to which order book depth is aggregated.

22% in the morning, drops to 12% and bounces back to 24% in the evening. If anything, the effects are stronger on the bid side. At a price impact of 5%, commonality is at a level of 16% in the morning, falls to 10% and rises again to 15% at the end of the day. These patterns are observed both in the R^2 values of the regressions as well as the coefficients (with the only exception of coefficients at 5% price impacts). Clearly, commonality varies over the day, starting out high, getting lower in the afternoon and increasing again at the end of the trading day. Independently from the level effects of firm liquidity, it exhibits the well-known U-shape over the trading day.

5.3 Market Momentum

While time-of-day effects are an explanation of variation in commonality over a short time horizon, market momentum is possible explanation for variation over a longer horizon. Numerous studies provide evidence that the correlation of stock returns is strongest in falling markets (see Conrad, Gultekin and Kaul (1991), Kroner and Ng (1998), Bekaert and Wu (2000), Longin and Solnik (2001) or Ang and Chen (2002)). If liquidity supply behaves in the same way, we would expect the comovement of liquidity to be stronger in falling markets. Therefore, we calculate the ten-day portfolio return of the our sample as a rolling window. The hightest return among the ten-day subsamples is 1.48% and the lowest is -4.60%. We choose these two subsamples and estimate Equation 2 separately for the rising market and the falling market.

Table 9 summarizes the results for the market momentum subsamples. If we compare the R^2 values for the upward and downward markets, we see that there is very strong evidence of a momentum effect: liquidity comoves far stronger in falling markets than in rising markets as expected. This momentum effect prevails despite the fact that the estimation includes the market return as an explanatory (albeit insignificant) variable. On the ask side, commonality is about one third higher in falling markets. For example, at a price impact of 0.5% the commonality in depth is about 10% in the rising market and 15% in the falling market. These figures increase to 21% in rising markets and 31% in falling markets for price impacts of 5%. Clearly, the momentum effect is stable with regard to the size of aggregate order book depth. On the bid side, the difference is even stronger: commonality in falling markets is about twice as high as in rising markets. For 0.5% impacts, the R^2 value is 6% in the upwards-trending sample and 15% in the downwards-trending sample. For 5%, the values are 12% and 21%.

6 Conclusion

In this paper, we depart from the observation that market-wide liquidity apparently gets priced, yet there is only very weak evidence that stock liquidity movement does actually exhibit a market-wide component. Extensive order book data from the Frankfurt Stock Exchange (FSE) in Germany enables us to measure common movements of the entire liquidity in the order book. We construct measures of order book liquidity and then estimate the sensitivity of firm liquidity to market liquidity. We alternatively use principal components analysis to doublecheck that our results are not biased.

In all, we observe strong evidence of commonalities in liquidity. In a reference scenario, we analyze commonality for liquidity at the best limit price. Although we document significant commonality, it is fairly weak (at 2%) as in the literature. However, once we take the liquidity supply in the order book into account, commonality increases dramatically. If the aggregated volume of all limit orders with price limits up to 1% from the best limit price is considered, commonality already increases to about 10%. If higher proportions of the liquidity in the book are taken into account, commonality can rise up to nearly 20%. A closer examination of the results also reveals that ask side commonalities are stronger than on the bid side. Measures of the price impact function lead to very similar results. Obviously, in a limit order market with a limit order book, the commonality of liquidity provision is drastically higher than the spread alone suggests. A closer examination also shows that commonality exhibits time-of-day effects that are independent of time-of-day effects in liquidity levels. Furthermore, commonality is much stronger in falling markets than in rising markets.

One implication of commonality is that an asset's liquidity will affect the investor's risk of holding the asset. An investor who holds a portfolio of stocks will not be able to eliminate liquidity risk. Further, commonality implies that assets will tend to be illiquid at the same time. This can potentially affect asset prices and lead to the instability of the market. Market-wide liquidity swings will tend to be stronger if a market is already trending downwards. Whether commonality really does reinforce market crises or whether effects stay confined to the asset class alone is an open question. However, events such as the 1987 stock market crash show that market-wide liquidity outflows can be substantial and can have a destabilizing impact.

In the light of our empirical findings, a natural question is to ask where commonality comes from. One hypothesis is that commonalities arise from the correlated trading behaviour of market participants. The most plausible determinants seem to be correlated liquidity demands, informed trading or discretionary trading. The theoretical literature leaves a lot of room for models to be developed in this area. Even in the absence of theory, some of these hypotheses should still be accessible to closer empirical examination. The principal components in the PCA are a further source of information as to the identity of the economic factors at work behind the commonality of liquidity.

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Based on the rebuilt order book, liquidity dynamics are examined. In contrats to findings for dealer markets, past market returns play a minor role in the determination of liquidity and liquidity commonality in Xetra, a pure limit order book market. Consequently, we provide evidence that liquidity provision by multiple sources in Xetra mitigates systemic liquidity risk introduced by the interrelation of return and liquidity. Keywords: limit order book, commonalities, liquidity, market microstructure JEL classication: G10, C32. âˆ—HÂ´elÂ´ena Beltran-Lopez is a FNRS research fellow at CORE, UniversitÂ´e catholique de Louvain, Voie du Ro-man Pays 34, B-1348 Louvain-la-Neuve, Belgium. Phone: +32 (0) 10 478160. Email: beltran@core.ucl.ac.be. Pierre Giot is Professor of Finance at the Department of Business Administration & CEREFIM, University of Namur, Rempart de la Vierge, 8, 5000 Namur, Belgium and CORE at UniversitÂ´e catholique de Louvain. Phone: +32 (0) 81 724887. Email: pierre.giot@fundp.ac.be.