1 Real Options and Investment under Uncertainty: An Overview

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1 Real Options: Main Ideas

The application of option concepts to value real assets has been an important and exciting growth area in the theory and practice of finance. It has revolutionized the way academics and practitioners think about investment projects by explicitly incorporating management flexibility into the analysis. This flexibility can represent a substantial part of the value of many projects. Neglecting it can grossly undervalue these investments and induce a miss-allocation of resources in the economy.[1]

The traditional approach to valuing investment projects, based on net present value (NPV), essentially involves discounting the expected net cash flows from a project at a discount rate that reflects the risk of those cash flows (the "risk-adjusted" discount rate). In this approach the adjustment for risk is made to the discount rate. An alternative approach is to make the adjustment for risk to the cash flows and to discount the resulting certainty-equivalent cash flows, instead of the expected cash flows, at the risk-free rate of interest.[2] The net-present-value approach is usually used in practice because it is thought normally to be easier to estimate the risk-adjusted discount rate than the certainty-equivalent cash flows.[3] However, in certain cases, such as in the case of commodities for which futures contracts exist, the certainty-equivalent cash flows are clearly easier to calculate since they can be obtained from future (or forward) prices.[4] So, when valuing projects in which the main uncertainty is the commodity price and future prices for the commodity exist, it is much easier to use the certainty-equivalent approach. Instead of having to obtain subjective forecasts of future spot prices of the commodity, which are highly volatile, market-traded future prices can be used. This approach bypasses the need to compute a risk-adjusted discount rate. Once the adjustment for risk has been appropriately made to the cash flows, the relevant discount rate is the risk-free rate of interest.

These results are more general than the above example might seem to indicate. Harrison and Kreps (1979), Harrison and Pliska (1981), and others have shown that, in perfect markets, the absence of arbitrage implies the existence of a probability distribution such that securities are priced based on their discounted (at the risk free rate) expected cash flows, where expectation is determined under this risk-neutral or risk-adjusted probability measure (also called the "equivalent martingale measure"). If markets are complete and all risks can be hedged, these probabilities are unique. Such would be the case, for example, when we are pricing a call option on a stock by forming a portfolio between the underlying stock and a riskless bond which dynamically replicates the payoff of the call option. If markets are not complete these risk-neutral probabilities are not necessarily unique, but any one of the (possibly infinite number of) probability distributions would determine the same market value of the security. Such would be the case, for example, when we are pricing a corporate bond subject to default risk when default can occur suddenly (with a certain probability per unit of time). When future contracts exist, futures prices are the expected spot prices at the maturity of the futures contract under this risk-neutral probability distribution. In the above discussion we have implicitly assumed that interest rates are constant, but these results also apply when interest rates are stochastic.

The critical advantage of working in this risk-neutral environment in which the relevant discount rate is the risk-free rate of interest is that it is an appropriate and convenient environment for option pricing. This allows the multiple operating options available in a typical investment project to be naturally incorporated in the analysis. These options include the optimal time to invest in a project, options to stop and restart production in response to price changes, the option to abandon the project if prices are too low to justify maintaining ongoing operations, the option to expand production, corporate growth options etc.

Option pricing theory, developed by Black and Scholes (1973), Merton (1973) and Cox and Ross (1976), introduced the concept of pricing securities by arbitrage methods. Since the option is valued *relative* to the underlying asset (and can in principle be replicated synthetically), it has the same value in the actual world as in a risk-neutral environment. For the purpose of valuing an option, *it can be assumed* that the expected rate of return on the underlying asset (as well as the option) is the risk-free rate of interest in such a risk-neutral environment. The expected value of the option at maturity, under the risk-neutral probability distribution, can then be discounted at the risk-free rate to obtain the current value of the option. If the market is complete the risk-neutral distribution is unique and can be obtained simply by replacing the actual (true) expected rate of return on the underlying asset by the risk-free rate of return.

Using this risk-neutral framework to value investment projects has three major advantages. First, it allows properly taking into account all the flexibilities (options) that the project might have. Second, it uses all the information contained in market prices (e.g., futures prices) when such prices exist. Third, it allows using the powerful analytical tools developed in contingent claims analysis to determine both the value of the investment project as well as its optimal operating policy (i.e., optimal exercise of the many real options the project might have).

These methods have been first successfully applied in the valuation of natural resource investments, such as gold and copper mines and oil deposits. A main reason for this is the existence of well-developed futures markets for these commodities from which essential market information can be extracted that makes use of the certainty-equivalent and risk-neutral approaches quite convenient.

In the first attempts to value investment projects in natural resources using this new approach (e.g., see Brennan and Schwartz 1985), the spot price of the commodity was assumed to follow geometric Brownian motion similar to the process assumed for stock prices in the option pricing literature. This allowed for straightforward extension of the option pricing framework to value real assets. Futures prices were used to determine the average convenience yield, which plays a similar role in the commodity spot price process as the dividend yield does in the stock price process.[5]

The stochastic process initially assumed for the spot commodity price, however, had some drawbacks. First, if the convenience yield is assumed constant, the model is unable to capture changes in the term structure of futures prices (for example, from backwardation to contango or vice-versa). In reality, the convenience yield experiences significant changes through time. Second, the model implies that the volatility of all futures returns is equal to the volatility of spot returns. The data shows, however, that the volatility of futures returns decreases with the time to maturity of the futures contract. Third, geometric Brownian motion implies that the variance of the distribution of spot prices grows linearly with time, whereas supply and demand adjustments to changing prices would suggest some type of mean reversion in spot commodity prices. In the last few years there have been several attempts to address the drawbacks of the basic natural-resource valuation model discussed above.[6,7]

In most capital investment situations, however, the sources of uncertainty in a project do not have futures prices from which to easily obtain the risk-adjusted process needed for valuation. In many cases the sources of uncertainty in the project are state variables that are not traded assets. Examples of this are product demand uncertainty, geological uncertainty, technological uncertainty, and cost uncertainty.

If a claim is contingent on the value of one or more state variables that are not traded assets, an equilibrium model of asset prices can be used to value the contingent claim. Generalizing Merton's (1973b) intertemporal capital asset pricing model, Cox, Ingersoll, and Ross (1985) derive a fundamental partial differential equation that must be satisfied by the value of all such general claims. This analysis implies that, for the purposes of pricing the contingent claim, a "risk-adjusted drift" for the stochastic process of the state variables can again be used such that the expected option payoff at maturity can be discounted at the riskless interest rate. In this case, the risk-adjusted drift is equal to the original drift minus an adjustment for risk (risk premium) that comes from the equilibrium model. The drift of the process and the risk adjustment from the equilibrium model now enter into the valuation model. In order to implement this approach it is necessary to know the correlation between changes in the state variables and aggregate wealth (the "beta").[8]

These general ideas can be applied to investment project valuation. For assets which do not have traded futures contracts or for other state variables that may affect output, such as uncertainty shocks on demand or costs, one must adjust the drift of the corresponding stochastic process using an equilibrium model to enable risk-free discounting. If a time series of the state variable is available to estimate the parameters of its stochastic process, it can be used to compute the correlation of changes in this state variable with the market portfolio. This correlation can then be used in an equilibrium model for determining the appropriate risk adjustment.

In practice, most real option problems must be solved using numerical methods. In many cases they can be modeled using partial differential equations (PDE's) and boundary conditions which the value of the project must satisfy. Their numerical solution gives not only the value of the project, but also the optimal strategy for exercising the options embedded in the analysis. The simplest real option problems involving one or two state variables can also be more conveniently solved using binomial or trinomial trees in one or two dimensions. But if the problems involve more state variables and/or are path-dependent, the more practical solution is to use Monte Carlo simulation methods. Until very recently simulation methods were not available for solving American-type options, which are the type typically encountered in real option problems. But in the last few years, methods have been developed which allow using simulation for solving American-style options.[9] For example, Longstaff and Schwartz (1998) developed a least-squares Monte Carlo approach to value American-type options by simulation. At every point in time the problem is to compare the value of immediate exercise with the conditional expected value (under the risk neutral measure) from continuation. The conditional expected value of continuation, for each path at each point in time, can be obtained from the fitted value of the linear regression of the discounted value (at the risk free rate) of the cash flows obtained from the simulation following the optimal policy in the future, on a set of basis functions of the state variables. Since this is a recursive procedure starting from the maturity of the option, the outcome is the optimal stopping time for each path in the simulation. Knowing the optimal stopping time for each path, the American option can then be easily valued. Chapter 27 by Cortazar reviews these developments and the use of numerical methods in general.

We pointed out a number of major advantages that real options valuation has over the traditional net present value approach. It explicitly allows for managerial flexibility in the form of options in the valuation procedure. It does not require the estimation of a risk-adjusted discount rate and uses the risk-free rate of interest as the discount rate. When market (e.g., future) prices exist, it avoids the need to make assumptions about the trajectory of spot prices in the future since it uses the information contained in futures prices.

Academic articles dealing with the application of option pricing theory to valuing real assets have appeared in the finance literature for more than fifteen years. The practical application of these ideas has mainly been taking place in the last several years. Although the methodology was first applied to natural resource investments, more recently we started seeing applications in a range of other areas, including research and development, development of new technologies, company valuation and M&As, intellectual property rights/intangible assets, etc. We predict that the real options approach to valuation will have a significant impact in the practice of finance and strategy over the next 5–10 years.

2 Real Options: Literature in Perspective

The sections that follow describe various stages in the development and evolution of the real options literature, organized around several broad themes. This classification should provide the reader with both a historical and a contextual perspective for the development of the ideas, problems, and techniques in real options analysis. The role and position in the literature of the selected readings in this book (with relevant chapter in parenthesis) is indicated when appropriate.

2.1 Underinvestment and Conceptual Options Approaches

The real options revolution arose in part as a response to the dissatisfaction of corporate practitioners, strategists, and some academics with traditional capital budgeting techniques. Well before the development of real options, corporate managers and strategists were grappling intuitively with the elusive elements of managerial operating flexibility and strategic interactions. Early critics (e.g., Dean [1951], Hayes and Abernathy [1980], Hayes and Garvin [1982]) recognized that standard discounted cash flow (DCF) criteria often undervalued investment opportunities, leading to myopic decisions, underinvestment and eventual loss of competitive position, because they either ignored or did not properly value important strategic considerations. Decision scientists further maintained that the problem lied in the application of the wrong valuation techniques altogether, proposing instead the use of simulation and decision tree analysis (see Hertz [1964], Magee [1964]) to capture the value of future operating flexibility associated with many projects. Proponents (e.g., Hodder and

Riggs [1985], Hodder [1986]) have argued that the problem rather arises from misuse of DCF techniques as commonly applied in practice. Myers (chapter 2), while confirming that part of the problem results from various misapplications of the underlying theory, acknowledges that traditional DCF methods have inherent limitations when it comes to valuing investments with significant operating or strategic options (e.g., in capturing the sequential interdependence among investments over time), suggesting that option pricing holds the best promise of valuing such investments. Trigeorgis and Mason (chapter 4) clarify that option valuation can be seen operationally as a special, economically-corrected version of decision tree analysis that is better suited in valuing a variety of corporate operating and strategic options. Baldwin and Clark (1992) discuss the importance of organizational capabilities in strategic capital investment, while Baldwin and Trigeorgis (1993) propose remedying the underinvestment problem and restoring competitiveness by developing specific adaptive capabilities viewed as an infrastructure for acquiring and managing real options.

Building on Myers's (1977) initial idea of thinking of discretionary investment opportunities as "growth options," Kester (chapter 3) conceptually discusses strategic and competitive aspects of growth opportunities. Dixit and Pindyck (chapter 5) and Trigeorgis (chapter 6) provide alternative conceptual real options frameworks for capital budgeting decisions. Other general conceptual frameworks are presented in Mason and Merton (1985), Trigeorgis and Mason (chapter 4), Brealey and Myers (2000), and Kulatilaka and Marcus (1988, 1992). Mason and Merton (1985), for example, provide a good discussion of many operating as well as financing options, and integrate them in a project financing for a hypothetical, large-scale energy project.

2.2 Review of Some Basic Models

The quantitative origins of real options derive from the seminal work of Black and Scholes (1973) and Merton (1973) in pricing financial options. Cox, Ross, and Rubinstein's (1979) binomial approach enabled a more simplified valuation of options in discrete-time. Margrabe (1978) values an option to exchange one risky asset for another, while Stulz (1982) analyzes options on the maximum (or minimum) of two risky assets and Johnson (1987) extends it to several risky assets. These papers opened up the potential to help analyze the generic option to switch among alternative uses and related options (e.g., abandon for salvage value or switch among alternative inputs or outputs). Geske (1979) values a compound option (i.e., an option to acquire another option), which in principle may be applied in valuing growth opportunities which become available only if earlier investments are undertaken. Carr (1988) combines the above two building blocks to value sequential (compound) exchange options, involving an option to acquire a subsequent option to exchange the underlying asset for another risky alternative. The above line of work opened up the potential, in principle, to value investments with a series of investment outlays that can be switched to alternative states of operation, and particularly to eventually help value strategic inter-project dependencies.

The book includes a number of papers as a review of some basic models more directly relevant to the valuation of capital investment opportunities or real options. Trigeorgis (chapter 7) reviews the basic principles of valuing various real options (e.g., to defer, expand, abandon) via simple binomial trees. Brennan and Schwartz (chapter 8) provide a clear exposition of natural resource valuation based on the certainty-equivalent approach using futures contracts. Dixit (chapter 9) uses real options theory to explain why firms often do not invest until price rises substantially above long-run average cost and do not exit a business for lengthy periods, sustaining operating losses, even after price falls substantially below this cost (hysteresis). Kulatilaka and Trigeorgis (chapter 10) present a simple, discrete-time model of the generic flexibility to switch between alternative technologies or operating project "modes", also illustrating a hysteresis effect where, even though immediate switching may seem attractive, it may be long-term optimal to wait. Pindyck (chapter 11) shows how to value in continuous time the option value of waiting under investment irreversibility and uncertainty, modeled using option pricing or dynamic programming.

2.3 Valuing Various Real Options

A number of seminal papers gave a boost to the real options literature by focusing on valuing quantitatively—in many cases deriving analytic, closed-form solutions—one type after another of a variety of real options, although each option was typically analyzed in isolation. The option to defer or initiate investment has been examined by McDonald and Siegel (chapter 12), by Paddock, Siegel, and Smith (chapter 37) in valuing offshore petroleum leases, and by Tourinho (1979) in valuing reserves of natural resources. Ingersoll and Ross (1992) reconsider the decision to wait in light of the beneficial impact of a potential future interest rate decline on project value. Majd and Pindyck (chapter 13) value the option to delay sequential construction for projects that take time to build, or there is a maximum rate at which investment can proceed. Carr (1988) and Trigeorgis (1993a) also deal with valuing sequential or staged (compound) investments. Trigeorgis and Mason (chapter 4), and Pindyck (chapter 15) examine options to alter (e.g., expand or contract) operating scale or capacity choice. The option to temporarily shut down and restart operations was analyzed by McDonald and Siegel (1985), and by Brennan and Schwartz (chapter 8).

Myers and Majd (chapter 14) analyze the option to permanently abandon a project for its salvage value seen as an American put option. Options to switch use (e.g., outputs or inputs) have been examined, among others, by Margrabe (1978), Kensinger (1987), Kulatilaka (1988), and Kulatilaka and Trigeorgis (chapter 10). Baldwin and Ruback (1986) show that future price uncertainty creates a valuable switching option that benefits short-lived projects. Future investment opportunities seen as corporate growth options are discussed in Myers (1977), Brealey and Myers (2000), Kester (chapter 3), Trigeorgis and Mason (chapter 4), Trigeorgis (chapter 6), Pindyck (chapter 15), and Chung and Charoenwong (1991).

Despite its enormous theoretical contribution, the focus of the earlier literature on valuing individual real options (i.e., one type of option at a time) has nevertheless limited its practical value. Real-life projects are often more complex in that they involve a collection of multiple real options, whose values may interact. An early exception is Brennan and Schwartz (chapter 8), who determine the combined value of the options to shut down (and restart) a mine, and to abandon it for salvage. They recognize that partial irreversibility resulting from costs of switching the mine's operating state may create a hysteresis or inertia effect making it long-term optimal to remain in the same operating state even if short-term cash flow considerations seem to favor early switching. Although hysteresis is a form of interaction between early and later decisions, however, Brennan and Schwartz do not explicitly address the interactions among individual option values. Trigeorgis (chapter 17) focuses explicitly on the nature of real option interactions pointing out, for example, that the presence of subsequent options can increase the value of the effective underlying asset for earlier options, while exercise of prior real options may alter (e.g., expand or contract) the underlying asset itself, and hence the value of subsequent options on it. Thus, the combined value of a collection of real options may differ from the sum of separate option values. Trigeorgis identifies conditions for when option interactions are small or large, negative or positive. Kulatilaka (1994) subsequently examines the impact of interactions among such options on their optimal exercise schedules. The recent recognition of real option interdependencies should enable a smoother transition from a theoretical stage to an application phase.

2.4 Strategy and Competition

An area of immense importance is that of competition and strategy. Sustainable competitive advantages resulting from patents, proprietary technologies, ownership of valuable natural resources, managerial capital, reputation or brand name, scale and market power, empower companies with valuable options to grow through future profitable investments and to more effectively respond to unexpected adversity or opportunities in a changing technological, competitive, or general business environment. A number of economists have addressed several competitive and strategic aspects of capital investment early on. For example, Roberts and Weitzman (1981) find that in sequential decision making, it may be worthwhile to undertake investments with negative NPV when early investment can provide information about future project benefits, especially when their uncertainty is greater. Baldwin (1982) finds that optimal sequential investment for firms with market power facing irreversible decisions may require a positive premium over NPV to compensate for the loss in value of future opportunities that results from undertaking an investment. Pindyck (chapter 15) analyzes options to choose capacity under product price uncertainty when investment is, again, irreversible. Dixit (1989) considers firm entry and exit decisions under uncertainty, showing that in the presence of sunk or costly switching costs it may not be long-term optimal to reverse a decision even when prices appear attractive in the short term. Kogut and Kulatilaka (chapter 36) analyze the international plant location option in the presence of mean-reverting exchange rate volatility.

From a more explicit real options perspective, a number of authors (e.g., Myers [1987], Kester [1984, 1993], Trigeorgis and Mason [1987], Trigeorgis [1988], Brealey and Myers [2000], and Trigeorgis and Kasanen [1991]) have initially dealt with competitive and strategic options rather conceptually. For example, Kester (chapter 3) develops qualitatively various competitive and strategic aspects of inter-project growth options, while Kester (1993) proposes a planned sequential, rather than parallel, implementation of a collection of interrelated consumer products when learning results from early product introductions (e.g., about available shelf space needed for similar subsequent products) and when competitive advantage is eroding. Trigeorgis and Kasanen (1991) also examine sequential project interdependencies and synergies as part of an ongoing strategic planning and control process. Kasanen (1993) also deals with the strategic problem of the interaction between current investments and future opportunities, using a spawning matrix structure to determine the optimal mix of strategic and operating projects.

In section IV of the book, Luehrman (chapter 18) discusses a conceptual framework for viewing strategy as managing a portfolio of real options. Strategic acquisitions of other companies also often involve a number of growth, divestiture, and other flexibility options, as discussed in Smith and Triantis (chapter 19). Childs, Ott, and Triantis (chapter 20) provide an intuitive and interesting discussion for managing portfolios of interrelated (e.g., R&D) projects.

More quantitatively, Trigeorgis (1991a) uses option pricing techniques to examine early investment that may preempt anticipated competitive entry, and to value the option to defer investment when impacted by random competitive entry (Trigeorgis, 1990b). Further departing from the common assumption of perfect competition, Smit and Trigeorgis (chapter 21) and Kulatilaka and Perotti (chapter 22) examine how the investment decisions of a firm will influence competitive reactions and the equilibrium market price or quantity when early investment generates a strategic (e.g., cost) advantage. Grenadier and Weiss (chapter 23) use option pricing to value investment in technological innovations. A simpler game-theoretic treatment of competitive reactions under different market structures in a real options framework is also given in Smit and Ankum (1993). Supplementing options analysis with game theoretic tools capable of incorporating strategic competitive counteractions promises to be an important and challenging direction for future research.

2.5 Numerical Techniques

In the more complex real-life option situations, such as those involving multiple interacting real options, analytic solutions may not exist and one may not even be always able to write down the set of partial differential equations describing the underlying stochastic processes. The ability to value such complex option situations has been enhanced, however, with various numerical analysis techniques, many of which take advantage of risk neutral valuation. Generally, there are two types of numerical techniques for option valuation: (1) those that approximate the underlying stochastic processes directly, and are generally more intuitive; and (2) those approximating the resulting partial differential equations. The first category includes various lattice approaches such as Cox, Ross and Rubinstein's (1979) standard binomial lattice and Trigeorgis's (chapter 24) log-transformed binomial method, which are particularly well suited to valuing complex projects with multiple embedded real options, a series of investment outlays, dividend-like effects, as well as option interactions; it also includes Monte Carlo simulation, initially used by Boyle (1977). Cortazar (chapter 27) reviews simulation techniques in the context of real options problems and applications in the context of broader numerical methods. Boyle (1988) shows how lattice frameworks can be extended to handle two state variables, while Hull and White (1988) suggest a control variate technique to improve computational efficiency when a similar derivative asset with an analytic solution is available. Examples of the second category include numerical integration, and implicit or explicit finite difference schemes used by Brennan (1979), Brennan and Schwartz (chapter 25), and Majd and Pindyck (1987). Finally, a number of analytic approximations are also available. A comprehensive review of such numerical techniques is given in the articles by Geske and Shastri (chapter 26), Trigeorgis (chapter 24), and Cortazar (chapter 27).

2.6 Applications

A variety of real options applications is provided in section VI, starting with Merton's (chapter 28) overview of options applications (part of his Nobel prize address). Nichols's (chapter 29) celebrated interview about real options applications at Merck is another natural apetizer. Kemna (chapter 30) describes actual cases involving the timing of developing an offshore oil field, valuing a growth option in a manufacturing venture, and the abandonment decision of a refining production unit.

In the area of *flexible manufacturing*, the flexibility provided by flexible manufacturing systems, flexible production technology or other machinery having multiple uses has been analyzed from an options perspective by Kulatilaka (1988, 1993), Triantis and Hodder (1990), Aggarwal (1991), and Kulatilaka and Trigeorgis (chapter 10), among others. Kulatilaka (chapter 31) values the flexibility of an actual dual-fuel industrial steam boiler over a rigid alternative. Baldwin and Clark (1993) studied the flexibility created by modularity in design that connects components of a larger system through standard interfaces.

Of course, early applications arose in the area of *natural resource investments* due to the availability of traded resource or commodity prices, high volatilities and long durations, resulting in higher and better option value estimates. Brennan and Schwartz (chapters 8 and 16) first utilized the convenience yield derived from futures and spot prices of a commodity to value the options to shut down or abandon a mine. Paddock, Siegel, and Smith (chapter 37) valued options embedded in undeveloped oil reserves and provided the first empirical evidence that option values are better than actual DCF-based bids in valuing offshore oil leases. Trigeorgis (chapter 32) values an actual minerals project considered by a major multinational company involving several options. Bjerksund and Ekern (chapter 33) value a Norwegian oil field with options to defer and abandon. Morck, Schwartz and Stangeland (1989) valued forestry resources under stochastic inventories and prices. Laughton and Jacoby (1993) studied biases in the valuation of certain commodity projects of different duration characterized by a mean-reverting price process rather than the standard random walk assumption.

In the area of *land development*, Titman (chapter 34), Quigg (chapter 38), and other authors have shown that the value of vacant urban land should reflect not only its value based on its best immediate use, but also its option value if development is delayed and the land is converted into its best alternative use in the future. In a different context, McLaughlin and Taggart (1992) view the opportunity cost of using excess capacity as the change in the value of the firm's options caused by diverting capacity to an alternative use. In *leasing*, a number of authors valued various operating options embedded in leasing contracts.

In the area of *large-scale energy projects and regulation*, Mason and Baldwin (1988) valued government subsidies to large-scale energy projects as put options, while Teisberg (1994) provides an option valuation analysis of investment choices by a regulated firm. In *research and development*, Kolbe, Morris, and Teisberg (1991) discuss option elements embedded in R&D projects. Option elements involved in the staging of *start-up ventures* are discussed in Sahlman (1998) and Trigeorgis (1993). Pindyck (chapter 35) discusses capital investments when the cost is uncertain in the context of a power plant application.

In *foreign investment*, Baldwin (1977) discusses various location, timing and staging options present when firms scan the global marketplace. Bell (1995) and Kogut and Kulatilaka (chapter 36), among others, examine entry, capacity, and switching options for firms with multinational operations under exchange rate volatility. Various other option applications are found in areas ranging from valuing mean-reverting cash flows in *shipping*, studied by Bjerksund and Ekern (1995), to *global warming* (e.g., Hendricks [1991]) and *environmental pollution compliance* options, analyzed by Edleson and Reinhardt (1995). The potential for future applications is clearly a growth option itself.

2.7 Empirical Evidence

Empirical evidence on the explanatory power of real options started to emerge only quite recently. Kester (chapter 3) estimates that the value of a firm's growth options is more than half the market value of equity for many firms, even 70–80% for more volatile industries. Similarly, Pindyck (chapter 15) also suggests that growth options represent more than half of firm value if demand volatility exceeds 0.2. An early application in valuing offshore petroleum leases and explaining market bids has been provided by Paddock, Siegel, and Smith (chapter 37). Quick (chapter 38) reports empirical results indicating that option-based land valuations are better approximations of market prices. Berger, Ofek, and Swary (chapter 39) provide evidence of market valuation of the abandonment option in plant closing decisions. Moel and Tufano (1999) provide evidence that mine operating decisions are consistent with real option theory. More evidence is clearly forthcoming. We hope that this work will help stimulate further applications and more evidence in the years ahead.

Notes

1. Calculating the values of multiple options embedded in investment projects separately and adding the results may lead to substantial overvaluation of project value, however.

2. The certainty-equivalent cash flows are the certain amounts which would have the same value as the uncertain cash flows.

3. Fama and French (1997) have recently emphasized the difficulties of accurately estimating risk-adjusted discount rates.

4. Here, we do not distinguish between forward and future prices, i.e., we assume that the interest rate is deterministic or that commodity prices are uncorrelated with interest rates.

5. The convenience yield is the flow of services that accrue to the holder of the spot commodity but not to the holder of a futures contract. In practice, the convenience yield is the adjustment needed in the drift rate of the spot price process to properly price existing futures contracts.

6. See for example Brennan (1991), Gibson and Schwartz (1990 and 1991), Ross (1997), Cortazar and Schwartz (1994), and Bessembinder, Coughenour, Seguin, and Smoller (1995).

7. Schwartz (1997) compares three models of the stochastic behavior of commodity prices in terms of their ability to price the term structure of futures prices and the term structure of futures return volatility. The first model is a one-factor model in which the log of the spot price of the commodity is assumed to follow a mean-reverting process. The second model assumes that the convenience yield is also stochastic and follows a mean-reverting process. In this model the convenience yield plays the role of a stochastic dividend in the spot price process. The third model extends the second by assuming also stochastic interest rates. For the two commercial commodities considered, copper and oil, the one-factor model does a poor job in explaining the characteristics of the data. The other two models, however, are able to capture many of the characteristics of the term structure of futures prices and volatilities. This type of approach is now being used to model the behavior of commodity prices.

8. For example, Brennan and Schwartz (1982) apply this framework to the valuation of a regulated firm in which the underlying state variable is the rate of return on the rate base.

9. See Longstaff and Schwartz (1988).

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Option to expand and option to delay are valued as call option while option to abandon are valued as put option. Decision tree analysis can be used as a basic framework to determine the value of options embedded in the investment project.Â This technique not only evaluates the research portfolio under uncertainty but also results in intuitive explanations for the various scenario paths.Â Industry Leaders Embracing Real Options. Industries using real options as a tool for strategic decision making started with oil and gas and mining companies and later expanded into utilities, biotechnology, and pharmaceuticals, and now into telecommunications, high-tech, and across all industries.