Gain of Directional Antennas
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Gain is an antenna property dealing with an antenna's ability to direct its radiated power in a desired direction, or synonymously, to receive energy preferentially from a desired direction. However, gain is not a quantity which can be defined in terms of physical quantities such as the Watt, ohm or joule, but is a dimensionless ratio. As a consequence, antenna gain results from the interaction of all other antenna characteristics. This article will explore these interactions using elementary definitions of antenna properties.

Antenna characteristics of gain, beamwidth and efficiency are independent of the antenna's use for either transmitting or receiving. Generally these characteristics are more simply described for the transmitting antenna; however, the properties described in this article apply to both cases.

Gain definitions, and antenna characteristics related to gain, are found in a glossary on page 10, and will appear in italics within text. First, the concept of directive gain will be examined, followed by related antenna factors such as beamwidth and efficiency. Some simple equations are listed at the conclusion which permit approximate computations of directive gain and half-power beamwidth for directional type antennas.

Directive Gain from a Hypothetical Antenna

An antenna does not amplify. It only distributes energy through space in a manner which can best make use of energy available. Directive gain is related to and is a measure of this energy distribution.

To visualize the concept of directive gain, assume an elastic sphere is filled with an incompressible medium having a shape as shown in Figure 1a. A dot at the center of the sphere represents a hypothetical *isotropic radiator* which has equal *radiation intensity* in all directions. Let the radius of the sphere be proportional to the power radiated by the isotropic radiator. Next, the sphere is deformed to create a new shape as shown in Figure 1b. As a result of our assumption that the sphere is filled with an incompressible medium, the volume must remain unchanged regardless of the change in shape; the sphere surface must bulge outward somewhere if another area of the surface is depressed.

For the surface shown in Figure 1b, the distance from the center dot to all points on the

sphere surface is no longer everywhere equal, although the average distance, which is equal to the original radius (r_o), remains the same. The distance from the center to a point on the deformed surface is now proportional to the radiation intensity in that direction. The ratio of the distance from the center to any particular point on the surface (r_d), to the average distance (or original sphere radius, r_o) is the *directive gain* in that direction. The value of the directive gain in the direction of its maximum value is the *directivity.*

a) Symmetric radiation pattern of an isotropic radiator.
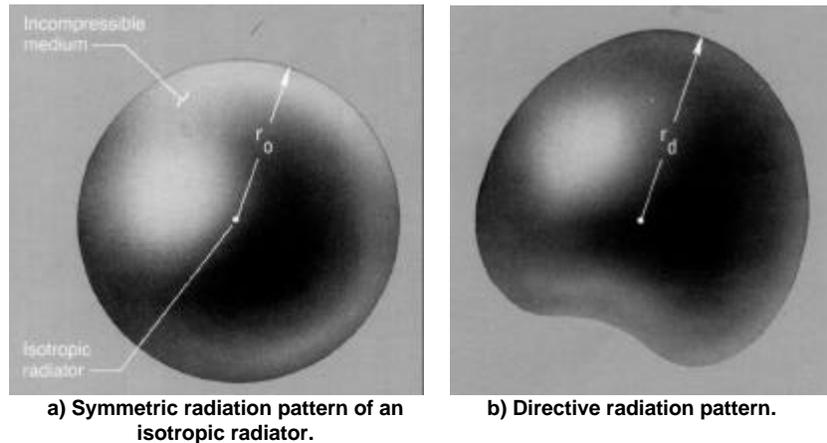b) Directive radiation pattern.

Figure 1. Directive gain resulting from the shape of the radiation pattern in a certain direction.

To accomplish this power distribution change, the hypothetical antenna at the sphere's center must be replaced by an antenna with the ability to direct radiated power in a desired direction. It is important to note that directive gain, as just described, is related only to the shape of the antenna's radiation pattern, and does not include efficiency factors.

Directive Gain and Beamwidth

An antenna's beamwidth is usually understood to mean the *half-power beamwidth,* that is, the angle between the two directions in which the directive gain of the major *radiation lobe* is one half the maximum value (one half the directivity), and is shown in Figures 2a, 2b, and 2c. Each curve represents the same antenna *radiation pattern,* but plotted to a different scale: in watts, voltage, and decibels(dB).
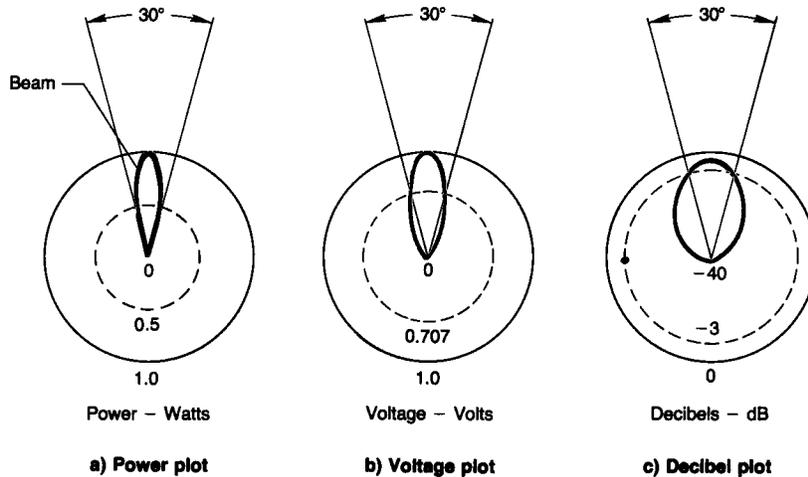
Figure 2. Equivalent half-power beamwidth representations of an antenna's radiation pattern.

For the power plot, the half-power beamwidth is measured at a value which is one half (.5) the peak of the *beam,* and is 30⁰ in the illustrated example. For the voltage plot, the half-power beamwidth is measured at a point which is .707 of the beam maximum .5 = .707^2), and is 30⁰. For the decibel plot, the half-power beam-width is 3dB from the beam maximum (10 $\log_{10}$ 0.5 = -3dB), and is 30⁰. Assuming that a significant amount of radiated power is not diverted into side lobes, then the directive gain is inversely proportional to beamwidth; as the beamwidth decreases, the directive gain increases.

A simplified approximation to an antenna's directive gain may be obtained by considering a convenient spherical-shaped boundary at which the power radiated by a hypothetical directional antenna can be measured. All power radiated from the hypothetical antenna may be imagined to flow outward and through the surface shown in Figure 3a.

This surface may be divided into square areas which are independent of radius, each occupying one degree in the vertical plane and one degree in the horizontal plane, and containing a total of 41,253 square degrees.* If all the power radiated by a directional *radiator* could be constrained to flow through one square degree, shown in Figure 3b, the directive gain in that direction would be 41,253 times the average directive gain. The directive gain for this power distribution is;

$$g_d = \frac{41,253}{1}$$

where all the power radiated is assumed to flow through an area of one square degree. Usually, directive gain is expressed in decibels, and for the directive gain just calculated, is equal to:

$$G_d = 10 \log_{10} g_d = 46 \text{ dB}.$$

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* 4π square radians (steradians) = 4π X (57.3)^2 square degrees = 41,253 square degrees.

a) Power flow through a convenient spherical boundary

b) Power flow through a square area of one square degree

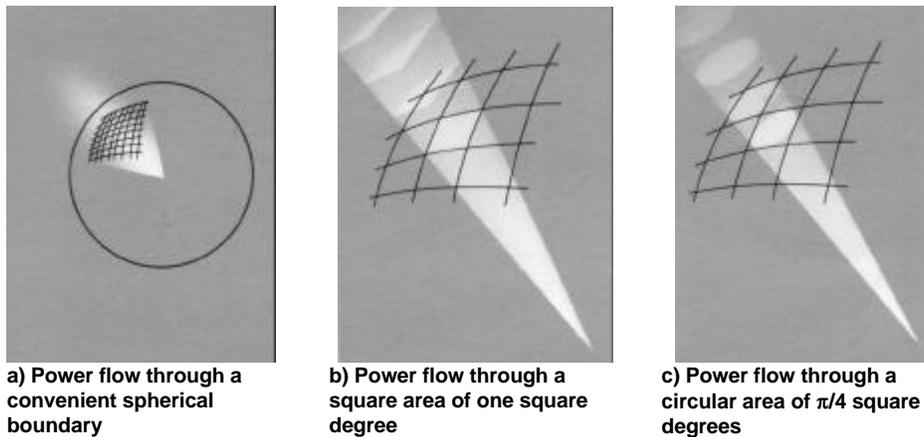c) Power flow through a circular area of $\pi/4$ square degrees

Figure 3. Simplified assumptions as to the shape of the radiated power yield approximate calculations of directive gain.

A more accurate approximation of the directive gain from the radiated pattern assumes that all the power radiated by a directional radiator is constrained to flow through an area which is circular in cross section, as shown in Figure 3c. Since the power radiated is constrained to flow through an area which is π /4.78% (as large. the resulting directive gain will be greater. and is given by:

$$g_d = \frac{41,253}{\theta_1 \theta_2} \cdot \frac{4}{\pi}$$

or

$$g_d = \frac{52,525}{\theta_1 \theta_2}$$

where θ_1 and θ_2 are orthogonal beamwidths, and represent the major and minor axis of the beam. For a circular beam shape, θ_1 is equal to θ_2.

In practical antenna applications, the beam is usually circular in cross section with many minor radiation lobes, or side lobes, present. To account for power flow in directions other than the beam's direction, an assumption is made that approximately 55% of the power radiated flows within the half-power beamwidth. The directive gain is now approximated by:

$$g_d = \frac{29,000}{\theta_1 \theta_2}$$

where θ_1 and θ_2 are the orthogonal half-power beamwidths of an asymmetric beam.

Although this last equation is very useful in obtaining an antenna's directive gain knowing the beamwidth, it must be remembered that it serves only as an approximation. The directive gain which results is based upon a radiation pattern exhibiting low-power losses in the side lobes. This is not always a good assumption. It

is possible for a radiation pattern to have the same beamwidth as for the 55% assumption, but have a large amount of power appear in the minor lobes. For example, if an additional 10% of the radiated power is lost to side lobe radiation, the directive gain is approximated by:

$$g_d = \frac{27,000}{\theta_1 \theta_2}$$

where it is now assumed that 45% of the radiated power flows through the half-power beamwidth. This last equation yields the most realistic value for the directive gain of reflector-type antennas. For horn-type antennas, it may be assumed that 60% of the power radiated flows within the beamwidth and the directive gain is:

$$g_d = \frac{31,000}{\theta_1 \theta_2}$$

Efficiencies Related to Power Gain, Realized Gain and Directive Gain

A quantity closely related to directive gain is *power gain,* g_p. For an ideal antenna with a *radiation efficiency* of 100%, directive gain is equal to power gain. For an antenna with losses (excluding reflection losses arising from impedance mismatch), power gain will be lower than directive gain, and is given by the equation:

$$G_p = g_d \, \eta$$

where η is the radiation efficiency, and is always less than unity.

Radiation efficiency is a measure of those losses internal to the antenna, such as I^2R losses in imperfect conductors and dielectrics. It is the ratio of the total power radiated by an antenna to the net power accepted by the antenna from a connected transmitter. Excluded from these losses is the power reflected back to the transmitter because of impedance mismatch. The implication is that an antenna tested for efficiency by the method described under the "gain measurements" paragraph to follow must be perfectly matched to the transmitter. This is a condition realizable under test conditions and at a single frequency, but is not a condition likely to exist under normal operating conditions, especially in a system which must operate over a wide frequency band.

When mismatch loss occurs, as it usually does, this loss must be subtracted from the power gain of the antenna to yield *realized gain.* Realized gain is important to the systems engineer, for it reveals how much signal will be available at the input to the receiver for a given field strength.

The *aperture* of an antenna is a planar surface near the antenna that is perpendicular to the direction of maximum radiation, and through which the major portion of the radiation passes. For parabolic reflector-type and horn-type antennas, the aperture is the

area of the paraboloid, or horn opening, respectively, as shown in Figure 4.
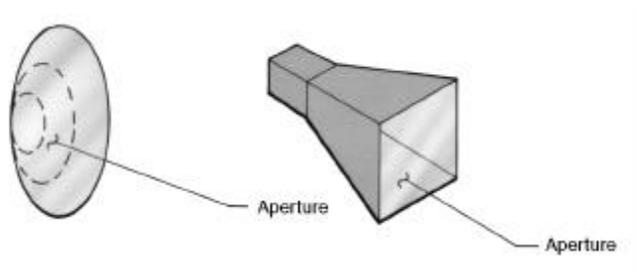
a) Parabolic reflector antenna   b) Horn antenna
Figure 4. Physical apertures of parabolic reflector- and horn-type antennas.

The manner in which energy is distributed over the aperture is referred to as *aperture illumination.* It is most simply explained by considering the field distribution over a parabolic reflector-horn feed antenna shown in Figure 5. For the aperture illumination shown in Figure 5a, a hypothetical feed produces equal radiation intensity over the angle subtended by the parabolic reflector, but with no energy spilled past the edges. Although this uniform aperture illumination is not achievable in practice, it is useful as a reference, as is the hypothetical isotropic radiator. The side lobes of the radiation pattern produced by uniform circular aperture illumination are approximately 18 dB lower in amplitude than the beam, which itself has as high a directive gain as can be achieved with a given aperture size.

Practical reflector-feed antennas, however, produce a tapered distribution of radiation intensity shown in Figure 5b. For this nonuniformly illuminated aperture, the radiation intensity at the edges of the aperture is approximately 10 dB less than at the center. As a result, the edges contribute less to the resultant, or secondary pattern, than the edges of the uniformly illuminated aperture. The side lobes of the radiation pattern produced are less in amplitude, and are more than 20 dB below the beam. However, the directive gain of this pattern is less than the uniformly illuminated aperture.
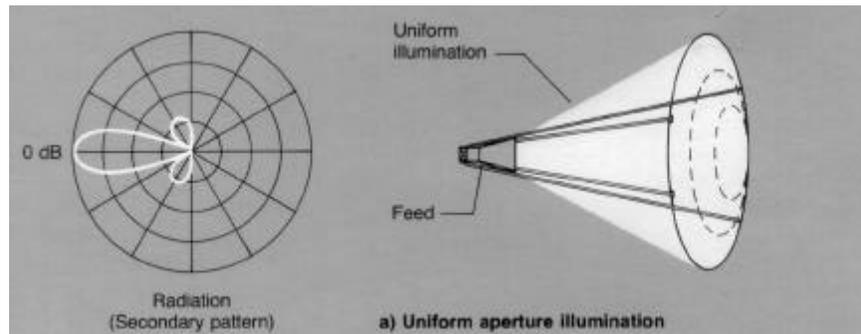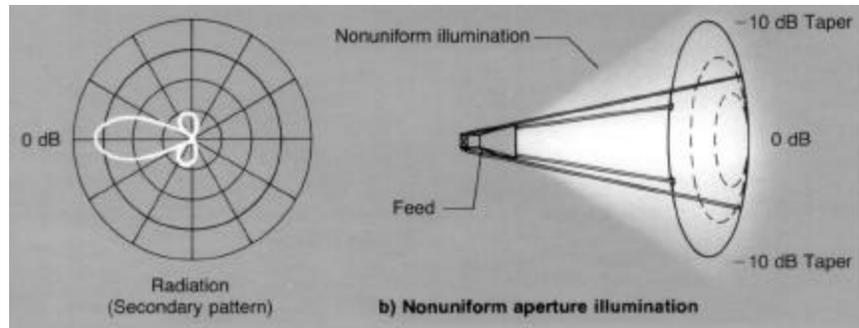
Figure 5. Field distributions, and radiation patterns produced, when a parabolic reflector's aperture is uniformly and nonuniformly illuminated.

The directive gains of the uniform and nonuniform illuminated apertures are related by *aperture illumination efficiency,* η_{ai} which is the ratio of the two directive gains, or

$$\eta_{ai} = \frac{g_d \text{ (nonuniform)}}{g_d \text{ (uniform)}}$$

It is possible, and in fact common, for the illumination taper across an aperture to be different for the feed pattern's orthogonal planes, particularly when the antenna must operate over a broad frequency range.

It is important to note that aperture illumination efficiency is related to directive gain, which, in turn, is related only to the shape of the radiation pattern and not to radiation efficiency. An antenna may simultaneously exhibit a low radiation efficiency and a high aperture illumination efficiency.

Antenna Efficiency—Aperture-Type Antennas

Antenna efficiency is concerned with the effectiveness of an antenna's aperture in directing, or collecting, radiated power. It is not related to radiation efficiency or mismatch loss, and need not be subtracted from directive gain.

Antenna efficiency is often sacrificed to obtain other desirable characteristics, such as a low side-lobe level, or wide bandwidth performance. For example, if it is necessary to illuminate a parabolic reflector with a horn feed over a band of frequencies, it is apparent the reflector's illumination will vary with frequency since a horn radiator's beamwidth is inversely proportional to frequency (or the aperture dimensions in terms of wavelength). Therefore, it is necessary to under-illuminate the reflector at the high-end frequency in order to not over-illuminate at the low-end frequency of the band.

Gain Measurements

The most generally used method for measuring an antenna's power gain is shown in Figure 6, and involves substituting a standard gain horn for the antenna under test and

comparing the power received by each. The power gain of the standard gain horn used as reference is computed from the horn's geometry. If the measurement is performed properly, which is extremely difficult to do, an accuracy approaching 0.1 dB is possible.

To measure realized gain, measurement for the antenna under test is made as it would be used in the field, with no special impedance matching, but with the standard gain horn always matched to the transmission line.

If the antenna under test is circularly polarized, the measurement becomes more complex, for there is no agreed-upon easily constructed gain standard that is circularly polarized and whose gain can be calculated from its geometry. Either specially designed reference antennas must be constructed and calibrated, or the antenna must be tested with reference to linear polarization (the standard gain horn) and suitably corrected for polarization mismatch. A discussion of these techniques is beyond the scope of this article.
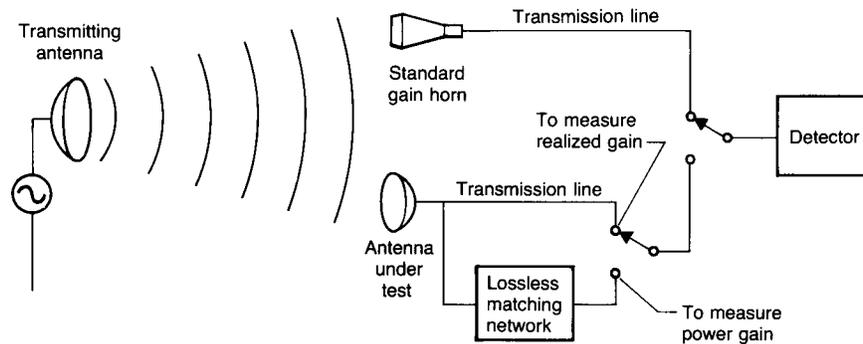

Figure 6. Power gain and realized gain measurements.

Optimum antenna performance is often a compromise between the conflicting requirements of maximum realized gain and beamwidth. The maximum possible realized gain is always desirable, of course, but the narrow beamwidth required to produce it requires precise positioning of the beam. Gain in the wrong direction is of little use.

Measurement of gain, difficult though it may be, is necessary to confirm that an antenna meets specification. Measured realized gain is the last word of performance, revealing the essence of the antenna, and is the most significant factor for any wireless link, be it the local TV station or the most exotic of spacecraft sending pictures of Mars to Earth.

Antenna Gain and Polarization

When antenna gain is specified or tested, generally the assumption made is that the polarization of the field is optimum—that is, the characteristic polarization of the antenna and the field in which it is measured, are the same. If the wave is polarized

differently from the antenna receiving it, then the power available at the antenna terminals will be less than maximum. Loss resulting from polarization mismatch can have any value between infinity and zero. Losses associated with some of the more common polarization mismatches are listed in Table 1.

		Field Polarization			
		Vertical ↑	Horizontal →	Right hand circular	Left hand circular
Antenna polarization	Vertical ↑	0 dB	∞	3 dB	3 dB
	Horizontal →	∞	0 dB	3 dB	3 dB
	Right hand circular	3 dB	3 dB	0 dB	∞
	Left hand circular	3 dB	3 dB	∞	0 dB

Table 1. Attenuation resulting from polarization mismatch between field and antenna.

Attenuation for the three polarizations listed is based on the polarization being either pure linear (vertical or horizontal) or pure circular. In practice, however, there is some coupling between orthogonal polarizations. If the polarizations are coincident, no attenuation (0 dB) occurs due to coupling mismatch between field and antenna. Polarizations which are either orthogonal linear or opposite-hand circular suffer infinite attenuation ( ) between field and antenna. Since a circular polarized wave can be re - solved into two equal vertical and horizontal components, each containing one half the total power radiated, only one half the power (3 dB) of a circularly polarized field is coupled to a linearly polarized antenna.

Gain Computations

Approximate solutions of beamwidth and directive gain for most directional type antennas can be obtained from the equations listed in Table 2. Also included is the approximate side-lobe level if the antenna is of the aperture-type shown. Side-lobe levels are not included in the equations for the uniformly illuminated apertures. Directive gain determined by either method should be used with caution; however, estimates of performance are adequate for preliminary system analysis.

Aperture-Type	Beamwidth (From Aperture)	Directive gain (From Aperture)	Directive gain (From Beamwidth)	Antenna Efficiency (Aperture Illumination Efficiency)
Uniformly illuminated circular aperture-hypothetical parabola ← a → 18 dB side-lobe level	$\theta = \dfrac{58\lambda}{a}$ $\theta = \theta_1 = \theta_2$	$g_d = \dfrac{15\,a^2}{\lambda^2}$ $g_d = \dfrac{9.87\,a^2}{\lambda^2}$	$g_d = \dfrac{52{,}525}{\theta^2}$ $\theta = \theta_1 = \theta_2$	100%
Uniformly illuminated rectangular aperture or linear array a │b 13 dB side-lobe level	$\theta_1 = \dfrac{51\lambda}{a}$ $\theta_2 = \dfrac{51\lambda}{b}$	$g_d = \dfrac{1.6\,a\,b}{\lambda^2}$	$g_d = \dfrac{41{,}253}{\theta_1\theta_2}$	100%
Rectangular horn a) Polarization plane: E-plane a_E 13 dB side-lobe level — — — — — — — — — b) Orthogonal polarization plane: H-plane ← a_H → 26 dB side-lobe level	$\theta_1 = \dfrac{56\lambda}{a_E}$ $\theta_2 = \dfrac{67\lambda}{a_H}$	$g_d = \dfrac{7.5\,a_E\,a_H}{\lambda}$	$g_d = \dfrac{31{,}000}{\theta_1\theta_2}$	60%
Nonuniformly illuminated circular aperture (10 dB taper)–normal parabola ← a → 26 dB side-lobe level	$\theta = \dfrac{72\lambda}{a}$ $\theta = \theta_1 = \theta_2$	$g_d = \dfrac{5\,a^2}{\lambda^2}$	$g_d = \dfrac{27{,}000}{\theta^2}$ $\theta = \theta_1 = \theta_2$	50%
	$a \gg \lambda$	$G_d = 10\,\log_{10}g_d$ dB	$G_d = 10\,\log_{10}g_d$ dB	

Table 2. Computations of directive gain end beamwldth for representative aperture-type

Selected Bibliography

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4. Ramo, S., J. R. Whinnery, Fields *and Waves in Modern Radio,* John Wiley & Sons, Inc., 1953. (Discussion of antenna gain with respect to half-wave dipole.)
5. Reich, H. J.. (ed.), *Very High-Frequency Techniques,* McGraw-Hill Book Co., New York, 1947. (Derivation of the equation for beamwidth in Chapter 1.)
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9. Wolff, E. A., *Antenna Analysis,* John Wiley & Sons, Inc., New York. (Discussion of gain in terms of admittance, current and effective aperture area.)

Glossary of Standard Antenna Terms

The "IEEE Standard Definitions of Term for Antennas" represent a consistent and comprehensive vocabulary suited for the effective communication and understanding of antenna theory. General use of these definitions of terms would eliminate much of the wide-spread inconsistency concerning antenna characteristics, particularly with regard to the basic parameters of gain, beamwidth, polarization and efficiency. For convenience, IEEE antenna terms used in this article are listed in this glossary.

Antenna efficiency of an aperture-type antenna. For an antenna with a specified planar aperture, the ratio at the maximum effective area of the antenna to the aperture area.

Aperture of an antenna. A surface, near or on an antenna, on which it is convenient to make assumptions regarding the field values for the purpose of computing fields at external points.
Note: The aperture Is often taken as that portion of a plane surface near the antenna, perpendicular to the direction of maximum radiation, through which the major part of the radiation passes.

Aperture illumination. The field over the aperture as described by amplitude, phase, and polarization distributions.

Aperture illumination efficiency. For a planar antenna aperture, the ratio of its directivity to the directivity obtained when the aperture illumination is uniform.

Beam. The major lobe of the radiation pattern.

Directive gain. In a given direction, 4π times the ratio of the radiation intensity in that direction to the total power radiated by the antenna.

Directivity. The value of the directive gain in the direction of its maximum value.

Effective area of an antenna. In a given direction, the ratio of power available at the terminals of a receiving antenna to the power per unit area of a plane wave incident on the antenna from that direction, polarized coincident with the polarization that the antenna would radiate.

Half-power beamwidth. In a plane containing the direction of the maximum of a beam, the angle between the two directions in which the radiation intensity is one half the maximum value of the beam.

Isotropic radiator. A hypothetical antenna having equal radiation intensity in all directions. Note: An isotropic radiator represents a convenient reference for expressing the directive properties of actual antennas.

Power gain. In a given direction, 4π times the ratio of the radiation intensity in that direction to the net power accepted by the antenna from the connected transmitter.

Notes: (1) When the direction is not stated, the power gain is usually taken to be the power gain in the direction of its maximum value.
(2) Power gain does not include reflection losses arising from mismatch of impedance.

Power gain in physical media. In a given direction and at a given point in the far field, the ratio of the power flux per unit area from an antenna to the power flux per unit area from an isotropic radiator at a specified location with the same power input as the subject antenna.
Note: The isotropic radiator must he within the smallest sphere containing the antenna. Suggested locations are antenna terminals and points of symmetry, if such exist.

Power gain referred to a specified polarization. The power gain of an antenna, reduced by the ratio of that portion of the radiation intensity corresponding to the specified polarization to the radiation intensity.

Radiation efficiency. The ratio of the total power radiated by an antenna to the net power accepted by the antenna from the connected transmitter.

Radiation, electromagnetic. The emission of energy in the form of electromagnetic waves.

Radiation intensity. In a given direction, the power radiated from an antenna per unit solid angle.

Radiation lobe. A portion of the radiation pattern bounded by regions of relatively weak radiation intensity.

Radiation pattern (antenna pattern). A graphical representation of the radiation properties at the antenna as a function of space coordinates.

Notes: (1) In the usual case the radiation pattern is determined in the far-field region and is represented as a function of directional coordinates. (2) Radiation properties include power flux density, field strength, phase, and polarization.

Radiator. Any antenna or radiating element that is a discrete physical and functional entity.

Realized gain. The power gain of an antenna in its environment, reduced by the losses due to the mismatch of the antenna input impedance to a specified impedance.

Realized radiation efficiency. The efficiency of an antenna in its environment reduced by all losses suffered by it, including: ohmic losses, mismatch losses, feedline transmission losses, and radome losses. (This term is not defined in the IEEE STD 145).

Relative power gain. The ratio of the power gain in a given direction to the power gain

of a reference antenna in its reference direction.

Note: Common reference antennas are half-wave dipoles, electric dipoles, magnetic dipoles, monopoles, and calibrated horn antennas.

Gain of Directional Antennas: WJ Tech Notes 1976. differently from the antenna receiving it, then the power available at the antenna terminals will be less than maximum. Loss resulting from polarization mismatch can have any value between infinity and zero. Since a circular polarized wave can be re solved into two equal vertical and horizontal components, each containing one half the total power radiated, only one half the power (3 dB) of a circularly polarized field is coupled to a linearly polarized antenna. Gain Computations Approximate solutions of beamwidth and directive gain for most directional type antennas can be obtained from the equations listed in Table 2. Also included is the approximate side-lobe level if the antenna is of the aperture-type shown.